Kinematics

A player on the Calgary Flames hockey team shoots the puck . . . and scores! A snowmobiler slides around an icy turn on a frozen lake against a biting cold wind. A rider drives a horse around a barrel at the Pro Rodeo barrel-racing competition at Writing-on-Stone, Alberta.

Every day, you see and experience motion. In this unit, you will learn the language of physics that is used to describe motion. You will then develop a set of equations and graphs that will help you describe and explain what happens when an object moves.

***e* WEB**

An exciting event at the Calgary Stampede is the fireworks display. To learn more about the physics of fireworks, follow the links at www.pearsoned.ca/school/physicssource.

Unit at a Glance

CHAPTER 1 Graphs and equations describe motion in one dimension.

1.1 The Language of Motion
1.2 Position-time Graphs and Uniform Motion
1.3 Velocity-time Graphs: Uniform and Non-uniform Motion
1.4 Analyzing Velocity-time Graphs
1.5 The Kinematics Equations
1.6 Acceleration due to Gravity

CHAPTER 2 Vector components describe motion in two dimensions.

2.1 Vector Methods in One Dimension
2.2 Motion in Two Dimensions
2.3 Relative Motion
2.4 Projectile Motion

Unit Themes and Emphases

- Change and Systems

Focussing Questions

The study of motion requires analyzing and describing how objects move. As you study this unit, consider these questions:

- How can knowledge of uniform and uniformly accelerated motion provide us with rules that predict the paths of moving objects and systems?
- How do the principles of kinematics influence the development of new mechanical technologies?

Unit Project

Are Amber Traffic Lights Timed Correctly?

- Using the kinematics equations learned in chapter 1, you will determine if the amber traffic lights in your area give drivers enough time to cross an intersection before the lights turn red.

CHAPTER

1

Graphs and equations describe motion in one dimension.

Key Concepts

In this chapter, you will learn about:

- scalar quantities
- vector quantities
- uniform motion
- uniformly accelerated motion

Learning Outcomes

When you have completed this chapter, you will be able to:

Knowledge

- define, qualitatively and quantitatively, displacement, velocity, and acceleration
- define operationally, compare and contrast scalar and vector quantities
- explain, qualitatively and quantitatively, uniform and uniformly accelerated motion when provided with written descriptions and numerical and graphical data

Science, Technology, and Society

- explain that the goal of science is knowledge about the natural world
- explain that the process for technological development includes testing and evaluating designs and prototypes on the basis of established criteria

Drivers participating in the annual Yukon Quest (Figure 1.1), a dogsled race between Whitehorse, Yukon Territory, and Fairbanks, Alaska, must complete a 1600-km course. Teams must constantly adjust to changing snow and ice conditions and battle prevailing winds as they race up and down snow- and ice-covered hills, through valleys, across rivers, and on to the finish line. Teams run six or more hours at a time, at an average speed of 13.0 km/h, in temperatures that can reach below -50 °C. In order to do well in such a race, teams need to pay attention to various motion details, such as position relative to the other teams, distances travelled, and times elapsed.

To successfully complete the Yukon Quest, a driver must know how a sled moves and how fast the dogs can run. Recording times in training logs and measuring distances helps drivers understand the motion of the sled and of their dog team. In this chapter, you will learn how to describe motion using the terms, graphs, and equations of the branch of physics called *kinematics*.

▲ **Figure 1.1** A Yukon Quest competitor experiences changes in position and velocity during a race.

1-1 QuickLab

Match a Motion

Problem

What types of motions can you generate using ticker tape?

Materials

clacker
power supply
ticker tape

Procedure

1. Measure and cut 4 or 5 lengths of ticker tape of length 40 cm (approximately).
2. Pull the ticker tape through the clacker at an even speed.
3. Repeat step 2 for a new length of ticker tape. For each new ticker tape, pull with a different kind of motion: quickly, slowly, speeding up, slowing down, etc.

Questions

1. Describe the spacing between dots on each ticker tape. Is it even, increasing, or decreasing?
2. Use the spacing between dots to describe the motion of the ticker tape. Is the ticker tape speeding up, slowing down, or constant?
3. What aspect of motion does the spacing between dots represent?
4. Have you covered all possible motions in the runs you did? If not, which ones did you omit? Give reasons why you omitted some types of motion.

Think About It

1. What is motion?
2. What types of motion can objects undergo?
3. What are some words used to describe motion?
4. How can you determine how far and how fast an object moves?
5. What does the term "falling" mean?
6. Describe the motion of falling objects.
7. Which falls faster: a heavy object or a light object?
8. How can a ball that's moving upward still be falling?

Discuss your answers in a small group and record them for later reference. As you complete each section of this chapter, review your answers to these questions. Note any changes in your ideas.

1.1 The Language of Motion

When describing motions, such as the ones in Figures 1.1 and 1.2, you can use many different expressions and words. The English language is rich with them. Many sports broadcasters invent words or expressions to convey action and to excite the imagination. Phrases such as "a cannonating drive from the point" or "blistering speed" have become commonplace in our sports lingo. In physics, you must be precise in your language and use clearly defined terms to describe motion. **Kinematics** is the branch of physics that describes motion.

kinematics: a branch of physics that describes motion

▲ **Figure 1.2** How would you describe the motions shown in these photos?

MINDS ON How Do Objects Move?

Study the photos in Figure 1.2. Describe the motion of the puck, the wheelchair, and the harpoon with respect to time. Jot down your descriptions and underline key words associated with motion. Compare your key words with those of a partner. Compile a class list on the chalkboard.

When describing motion, certain words, such as speed, acceleration, and velocity, are common. These words have slightly different meanings in physics than they do in everyday speech, as you will learn in the following subsection.

Physics Terms

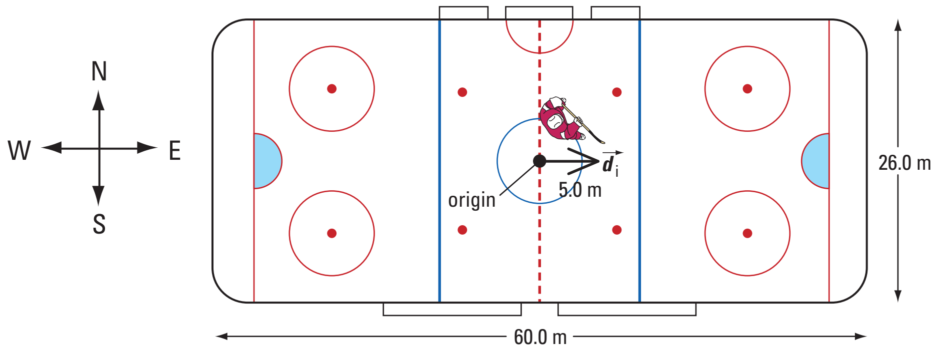
origin: a reference point

position: the straight-line distance between the origin and an object's location; includes magnitude and direction

scalar quantity: a measurement that has magnitude only

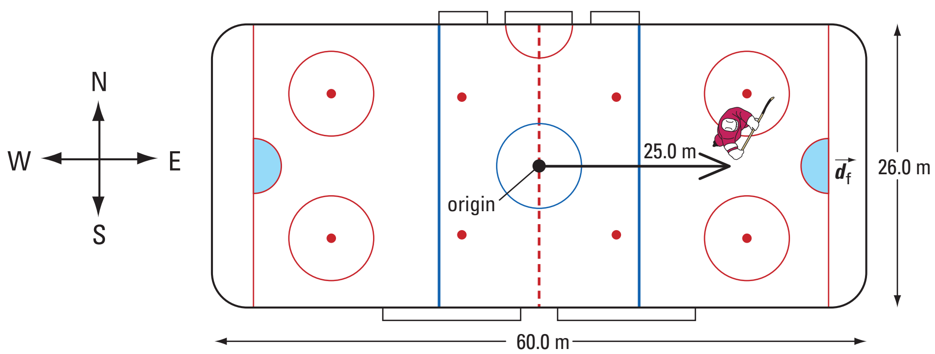
vector quantity: a measurement that has both magnitude and direction

It's Saturday night and the Edmonton Oilers are playing the Calgary Flames. In order to locate players on the ice, you need a reference system. In this case, select the centre of the ice as the reference point, or **origin**. You can then measure the straight-line *distances*, d, of players from the origin, such as 5.0 m. If you specify a direction from the origin along with the distance, then you define a player's **position**, $\vec{d}$, for example, 5.0 m [E] (Figure 1.3). The arrow over the variable indicates that the variable is a vector quantity. The number and unit are called the *magnitude* of the vector. Distance, which has a magnitude but no direction associated with it, is an example of a **scalar quantity**. **Vector quantities** have both magnitude and direction. Position is an example of a vector quantity.



▲ **Figure 1.3** The player's *position* is 5.0 m [east of the origin] or simply 5.0 m [E]. The player is at a *distance* of 5.0 m from the origin.

If the player, initially 5.0 m [east of the origin], skates to the east end of the rink to the goal area, his position changes. It is now 25.0 m [east of the origin] or 25.0 m [E] (Figure 1.4). You can state that he has travelled a straight-line distance of 20.0 m, and has a displacement of 20.0 m [E] relative to his initial position.



▲ **Figure 1.4** The player's *position* has changed. A change in position is called *displacement*.

Distance travelled is the length of the path taken to move from one position to another, regardless of direction. **Displacement**, $\Delta\vec{d}$, is the change in position. The player's displacement is written as

$$\Delta\vec{d} = 20.0 \text{ m [E]}$$

where Δ is the Greek letter delta that means "change in." Calculate the change in a quantity by subtracting the initial quantity from the final quantity. In algebraic notation, $\Delta R = R_f - R_i$. You can calculate the displacement of the player in the following manner:

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i \\ &= 25.0 \text{ m [E]} - 5.0 \text{ [E]} \\ &= 20.0 \text{ m [E]}\end{aligned}$$

PHYSICS INSIGHT

Technically, if you are standing away from the origin, you are displaced a certain distance and in a certain direction. However, the Δ sign is not used with position unless the object you are referring to has moved from the origin to its current position, that is, unless the object has experienced a *change* in position.

distance: the length of the path taken to move from one position to another

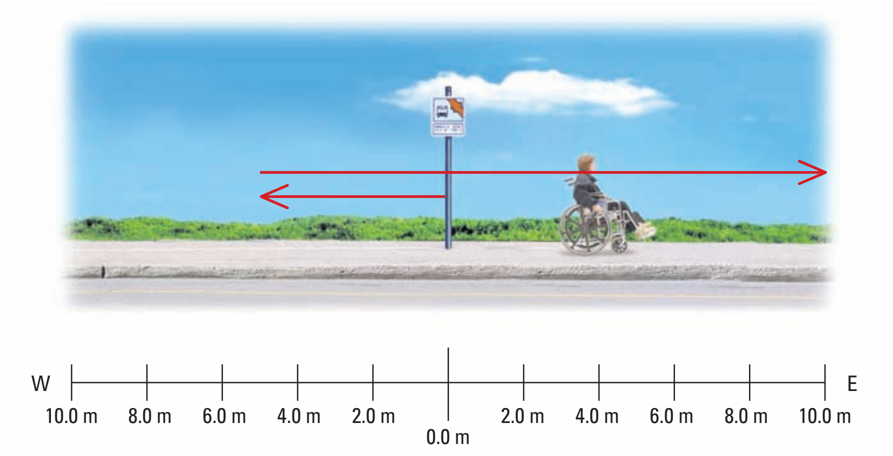
displacement: a straight line between initial and final positions; includes magnitude and direction

info **BIT**

Pilots use radar vectors when landing their aircraft. Radar vectors are instructions to fly in a particular direction and usually include altitude and speed restrictions.

Sign Conventions

How would you determine your final distance and displacement if you moved from a position 5.0 m [W] to a position 10.0 m [E] (Figure 1.5)?



▲ **Figure 1.5** The person travels a distance of 5.0 m + 10.0 m = 15.0 m. What is the person's displacement? What is the person's final position relative to the bus stop?

To calculate the distance travelled in the above scenario, you need only add the magnitudes of the two position vectors.

$$\begin{aligned}\Delta d &= 5.0\text{ m} + 10.0\text{ m}\\ &= 15.0\text{ m}\end{aligned}$$

To find displacement, you need to *subtract* the initial position, $\vec{d}_i$, from the final position, $\vec{d}_f$. Let $\vec{d}_i$ = 5.0 m [W] and $\vec{d}_f$ = 10.0 m [E].

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i\\ &= 10.0\text{ m [E]} - 5.0\text{ m [W]}\end{aligned}$$

Note that subtracting a vector is the same as adding its opposite, so the negative west direction is the same as the positive east direction.

$$\begin{aligned}\Delta\vec{d} &= 10.0\text{ m [E]} - 5.0\text{ m [W]}\\ &= 10.0\text{ m [E]} + 5.0\text{ m [E]}\\ &= 15.0\text{ m [E]}\end{aligned}$$

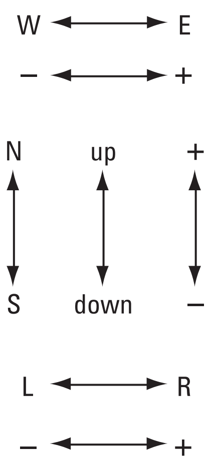
Another way of solving for displacement is to designate the east direction as positive and the west direction as negative (Figure 1.6). The two position vectors become $\vec{d}_i$ = 5.0 m [W] = −5.0 m and $\vec{d}_f$ = 10.0 m [E] = +10.0 m. Now calculate displacement:

$$\begin{aligned}\Delta\vec{d} &= \vec{d}_f - \vec{d}_i\\ &= +10.0\text{ m} - (-5.0\text{ m})\\ &= +15.0\text{ m}\end{aligned}$$

Since east is positive, the positive sign indicates that the person has moved 15.0 m east. Practise finding position and displacement in the next Skills Practice and example.

PHYSICS INSIGHT

When doing calculations with measured values, follow the rules on rounding and the number of significant digits. Refer to pages 876–877 in this book.



▲ **Figure 1.6** Let east be positive and west negative. Similarly, north, up, and right are usually designated as positive.

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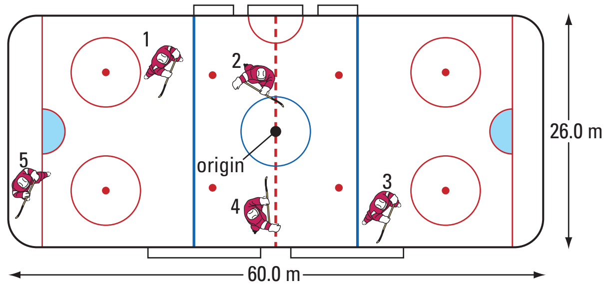
On April 26, 2004, Stephane Gras of France did 445 chin-ups in one hour. If you consider up as positive, then Gras made 445 positive displacements and 445 negative displacements, meaning that his net displacement was zero!

SKILLS PRACTICE Finding Position and Displacement



▲ **Figure 1.7**

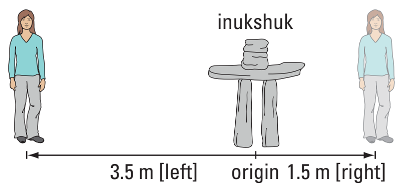
1. Create a scale using the dimensions of the hockey rink (Figure 1.7). Measuring from the centre of the player's helmet,
   (a) find each player's position relative to the north and south sides of the rink.
   (b) find each player's position relative to the east and west sides of the rink.
   (c) If the player moves from position 2 to position 4 on the rink, what is his displacement?

Example 1.1

A traveller initially standing 1.5 m to the right of the inukshuk moves so that she is 3.5 m to the left of the inukshuk (Figure 1.8). Determine the traveller's displacement algebraically
(a) using directions (b) using plus and minus signs

Given
$\vec{d}_i$ = 1.5 m [right]
$\vec{d}_f$ = 3.5 m [left]

Required
displacement ($\Delta\vec{d}$)



▲ **Figure 1.8**

Analysis and Solution
To find displacement, use the equation $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$.

(a) $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$
$= 3.5\text{ m [left]} - 1.5\text{ m [right]}$
$= 3.5\text{ m [left]} - (-1.5\text{ m [left]})$
$= 3.5\text{ m [left]} + 1.5\text{ m [left]}$
$= 5.0\text{ m [left]}$

(b) Consider right to be positive.
$\vec{d}_i = 1.5\text{ m [right]} = +1.5\text{ m}$
$\vec{d}_f = 3.5\text{ m [left]} = -3.5\text{ m}$

$\Delta\vec{d} = \vec{d}_f - \vec{d}_i$
$= -3.5\text{ m} - (+1.5\text{ m})$
$= -3.5\text{ m} - 1.5\text{ m}$
$= -5.0\text{ m}$

The answer is negative, so the direction is left.

Paraphrase
The traveller's displacement is 5.0 m [left] of her initial position.

Note that the direction of *displacement* is relative to initial position, whereas the direction of *position* is relative to the designated origin, in this case, the inukshuk.

Practice Problems

1. Sprinting drills include running 40.0 m [N], walking 20.0 m [N], and then sprinting 100.0 m [N]. What is the sprinter's displacement from the initial position?
2. To perform a give and go, a basketball player fakes out the defence by moving 0.75 m [right] and then 3.50 m [left]. What is the player's displacement from the starting position?
3. While building a wall, a bricklayer sweeps the cement back and forth. If she swings her hand back and forth, a distance of 1.70 m, four times, calculate the distance and displacement her hand travels during that time.

Answers
1. 160.0 m [N]
2. 2.75 m [left]
3. 6.80 m, 0 m

▲ **Figure 1.9** Scalar or vector?

For all subsequent problems in this book, you will be using plus and minus signs to indicate direction. This method is more flexible for problem solving and easier to use.

Like distance and displacement, speed and velocity is another scalar-vector pair. *Speed* is the rate at which an object moves. It is a scalar quantity, so it has magnitude only; for example, $v = 50$ km/h (Figure 1.9). *Velocity* is a vector quantity, so it has both magnitude (speed) and direction. If you are travelling south from Fort McMurray to Lethbridge at 50 km/h, your velocity is written as $\vec{v} = 50$ km/h [S]. If you designate south as negative, then $\vec{v} = -50$ km/h. *Acceleration* is a vector quantity that represents the rate of change of velocity. You will study aspects of displacement, velocity, and acceleration, and their interrelationships, in the sections that follow.

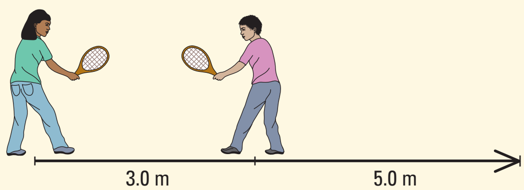
1.1 Check and Reflect

Knowledge

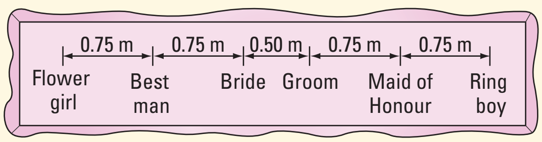
1. What two categories of terms are used to describe motion? Give an example of each.
2. Compare and contrast distance and displacement.
3. What is the significance of a reference point?

Applications

4. Draw a seating plan using the statements below.
   (a) Chad is 2.0 m [left] of Dolores.
   (b) Ed is 4.5 m [right] of Chad.
   (c) Greg is 7.5 m [left] of Chad.
   (d) Hannah is 1.0 m [right] of Ed.
   (e) What is the displacement of a teacher who walks from Greg to Hannah?
5. A person's displacement is 50.0 km [W]. What is his final position if he started at 5.0 km [E]?
6. Using an autuk (a type of sealskin racquet), two children play catch. Standing 3.0 m apart, the child on the right tosses the ball to the child on the left, and then moves 5.0 m [right] to catch the ball again. Determine the horizontal distance and displacement the ball travels from its initial position (ignore any vertical motion).



7. Below is a seating plan for the head table at a wedding reception. Relative to the bride, describe the positions of the groom, best man, maid of honour, and flower girl.



eTEST

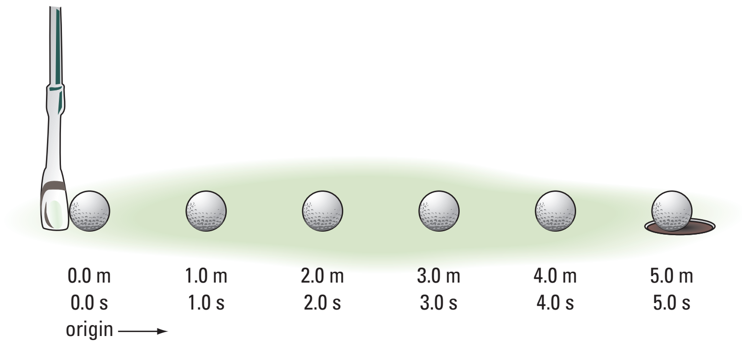
To check your understanding of scalar and vector quantities, follow the eTest links at www.pearsoned.ca/school/physicssource.

1.2 Position-time Graphs and Uniform Motion

You are competing to win the Masters Golf Tournament. The hole is 5.0 m away (Figure 1.10). You gently hit the ball with your club and hold your breath. Time seems to stop. Then, 5.0 s later, it rolls into the hole. You have won the tournament!

From section 1.1, you know that displacement is the change in an object's position. If you replay the sequence of motions of your winning putt in 1.0-s intervals, you can measure the displacements of the golf ball from you, the putter, to the hole (Figure 1.11).

▲ **Figure 1.10** You can represent motion in sports using vectors and graphs.

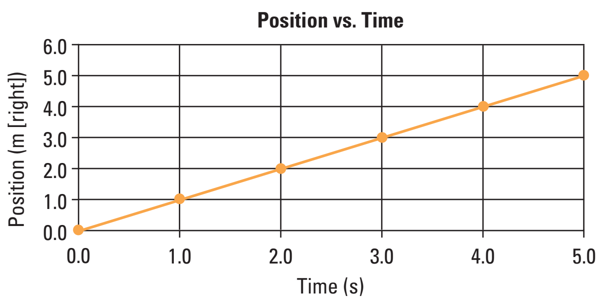


▲ **Figure 1.11** What is the golf ball's displacement after each second?

Table 1.1 displays the data from Figure 1.11 for the golf ball's position from you at 1.0-s intervals. By graphing the data, you can visualize the motion of the golf ball more clearly (Figure 1.12).

▼ **Table 1.1** Position-time data

	Time (s)	Position (m [right])
t_0	0.0	0.0
t_1	1.0	1.0
t_2	2.0	2.0
t_3	3.0	3.0
t_4	4.0	4.0
t_5	5.0	5.0



▲ **Figure 1.12** A position-time graph of the golf ball

Velocity

Notice that the graph in Figure 1.12 is a straight line. A straight line has a constant slope. What does constant slope tell you about the ball's motion? To answer this question, calculate the slope and keep track of the units. Designate toward the hole, to the right, as the positive direction.

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Practise calculating average speed and average velocity. Go to www.pearsoned.ca/school/physicssource.

Recall that slope $= \frac{\text{rise}}{\text{run}}$. For position–time graphs, this equation becomes slope $= \frac{\text{change in position}}{\text{change in time}}$

A change in position is displacement. So, the equation for slope becomes

$$\begin{aligned} \text{slope} &= \frac{\Delta \vec{d}}{\Delta t} \\ &= \frac{\vec{d}_f - \vec{d}_i}{t_f - t_i} \\ &= \frac{+5.0 \text{ m} - 0.0 \text{ m}}{5.0 \text{ s} - 0.0 \text{ s}} \\ &= +1.0 \text{ m/s} \end{aligned}$$

The answer is positive, so the golf ball moves at a rate of 1.0 m/s [right].

Notice that the units are m/s (read metres per second). These units indicate speed or velocity. Since displacement is a vector quantity, the slope of the position-time graph in Figure 1.12 gives you the **velocity**, $\vec{v}$, of the ball: the change in position per unit time. Because you have calculated velocity over a time interval rather than at an instant in time, it is the *average velocity*.

PHYSICS INSIGHT

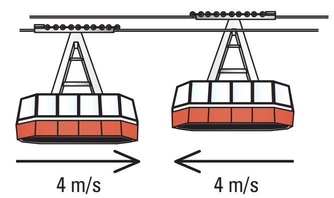
Speed has magnitude only. Velocity has both magnitude and direction.

velocity: rate of change in position

$$\vec{v} = \frac{\Delta \vec{d}}{\Delta t}$$

Speed and Velocity

Objects travelling at the same speed can have different velocities. For example, a tram carries passengers across a ravine at a constant speed. A passenger going to the observation deck has a velocity of 4 m/s [right] and a passenger leaving the deck has a velocity of 4 m/s [left] (Figure 1.13). Their speeds are the same, but because they are travelling in opposite directions, their velocities are different.



▲ **Figure 1.13** Objects with the same speed can have different velocities.

1-2 Decision-Making Analysis

Traffic Safety Is Everyone's Business

The Issue

In an average year in Alberta, traffic accidents claim six times more lives than homicide, eight times more lives than AIDS, and 100 times more lives than meningitis. Collisions represent one of the greatest threats to public safety.

Background Information

In the Canadian 2002 Nerves of Steel: Aggressive Driving Study, speeding was identified as one of two common aggressive behaviours that contribute to a significant percentage of all crashes. The Alberta Motor Association's Alberta Traffic Safety Progress Report has suggested that a province-wide speed management program could significantly improve levels of road safety, decreasing both speed and casualties. One suggested program is the implementation of the vehicle tachograph, a device required in Europe to improve road safety.

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To learn more about how speeding is a key contributing factor in casualty collisions in Alberta, follow the links at www.pearsoned.ca/school/physicssource.

Analysis

Your group has been asked to research different traffic safety initiatives. The government will use the results of your research to make the most appropriate decision.

1. Research
   (a) how state- or province-wide speed management programs have influenced driver behaviour
   (b) the societal cost of vehicle crashes
   (c) driver attitudes toward enforcement of and education about traffic safety issues
2. Analyze your research and decide which management program should be used.
3. Once your group has completed a written report recommending a particular program, present the report to the rest of the class, who will act as representatives of the government and the community.

So far, you have learned that the slope of a position-time graph represents a rate of change in position, or velocity. If an object moves at constant velocity (constant magnitude and direction), the object is undergoing **uniform motion**.

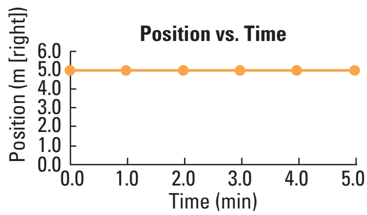
A position-time graph for an object **at rest** is a horizontal line (Figure 1.14). An object at rest is still said to be undergoing uniform motion because its change in position remains constant over equal time intervals.

uniform motion: constant velocity (motion or rest)

at rest: not moving; stationary

Concept Check

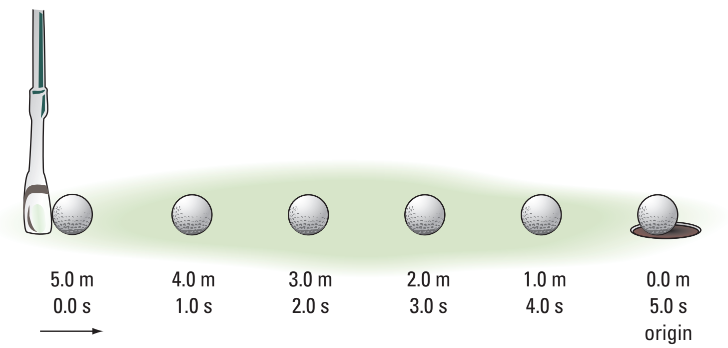
(a) Describe the position of dots on a ticker tape at rest. What is the slope of the graph in Figure 1.14?
(b) Describe the shape of a position-time graph for an object travelling at a constant velocity. List three possibilities.



▲ **Figure 1.14** A position-time graph for a stationary object

Frame of Reference

If you were to designate the hole, rather than the putter, as the origin (starting point) in the golf tournament (Figure 1.15(a)), your data table would start at 5.0 m [left] at time 0.0 s, instead of at 0.0 m and 0.0 s (Table 1.2). The values to the left of the hole are positive.



▲ **Figure 1.15(a)** Designating an origin is arbitrary. In this example, the hole is the origin and all positions are measured relative to it.

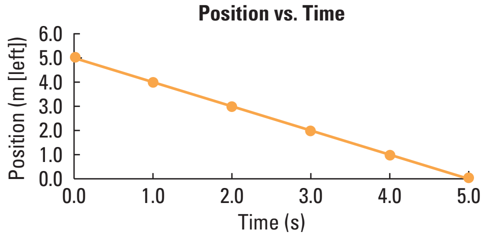
▼ **Table 1.2** Position-time data

	Time (s)	Position (m [left])
t_0	0.0	5.0
t_1	1.0	4.0
t_2	2.0	3.0
t_3	3.0	2.0
t_4	4.0	1.0
t_5	5.0	0.0

info BIT

On May 31, 2004 in Moscow, Ashrita Furman of the USA walked 1.6 km while continuously hula-hooping in 14 min 25 s. He also holds the world record for the fastest time for pushing an orange with his nose. On August 12, 2004, he pushed an orange 1.6 km in 24 min 36 s. What was his speed, in km/h and m/s, for each case? (See Unit Conversions on page 878.)

The corresponding position-time graph is shown in Figure 1.15(b).



▲ **Figure 1.15(b)** If you change your reference frame, the position-time graph also changes. Compare this graph with the graph in Figure 1.12.

From the graph,

$$\begin{aligned}
\text{slope} &= \vec{v} \\
&= \frac{\Delta \vec{d}}{\Delta t} \\
&= \frac{\vec{d}_{\mathrm{f}} - \vec{d}_{\mathrm{i}}}{t_{\mathrm{f}} - t_{\mathrm{i}}} \\
&= \frac{0.0\ \text{m} - (+5.0\ \text{m})}{5.0\ \text{s} - 0.0\ \text{s}} \\
&= -1.0\ \text{m/s}
\end{aligned}$$

The velocity of the golf ball is −1.0 m/s. What does the negative sign mean? It means the ball is travelling opposite to the direction to which the positions of the ball are measured. It does not mean that the golf ball is slowing down. Since positions, now measured to the left of the hole (the new origin) are designated positive, any motion directed to the right is described as being negative. In this case, you can also see that the ball is decreasing its position from the origin with increasing time. The ball travels to the right toward the hole, decreasing its position each second by 1.0 m — it travels at 1.0 m/s to the right or −1.0 m/s [left] as indicated by the downward slope on the graph.

Concept Check

Determine how the velocity of the golf ball can be positive if the hole is at the origin.

e WEB

In November 2004, at an altitude of 33 000 m, the X-43A recorded a speed of Mach 9. Use the Internet or your local library to research the term "Mach" as used to describe the speed of an object. How did this term originate? What is the difference between Mach and ultrasonic? Write a brief summary of your findings. To learn more about Mach, follow the links at www.pearsoned.ca/school/physicssource.

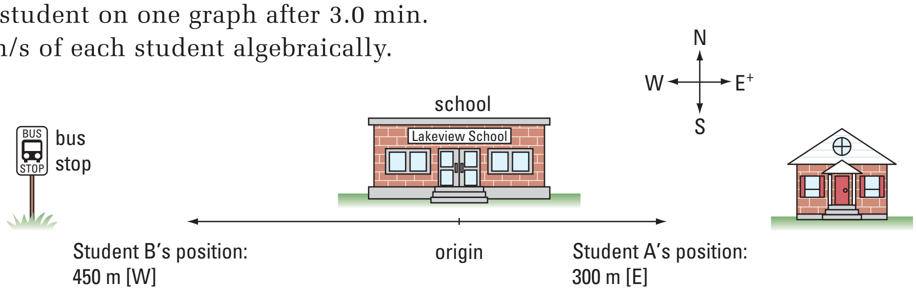
Below is a summary of what you have learned:
- The slope of a position-time graph represents velocity.
- The velocity is the *average* velocity for the time interval.
- Your choice of reference frame affects the direction (sign) of your answer.
- A straight line on a position-time graph represents uniform motion.

Comparing the Motion of Two or More Objects on a Position-time Graph

You can represent the motions of two objects on one graph, as long as the origin is the same for both objects. You can then use the graph to compare their motions, as in the next example.

Example 1.2

At the end of the school day, student A and student B say goodbye and head in opposite directions, walking at constant rates. Student B heads west to the bus stop while student A walks east to her house. After 3.0 min, student A is 300 m [E] and student B is 450 m [W] (Figure 1.16).
(a) Graph the position of each student on one graph after 3.0 min.
(b) Determine the velocity in m/s of each student algebraically.



▲ **Figure 1.16**

Given

Choose east to be positive.

$\Delta\vec{d}_A = 300 \text{ m [E]} = +300 \text{ m}$

$\Delta\vec{d}_B = 450 \text{ m [W]} = -450 \text{ m}$

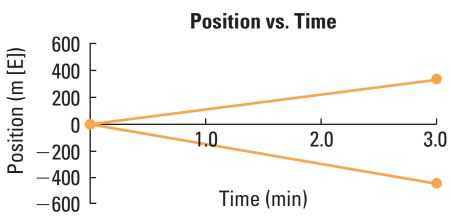
$\Delta t = 3.0 \text{ min}$

Required

(a) position–time graph
(b) velocity ($\vec{v}_A$ and $\vec{v}_B$)

Analysis and Solution

(a) Since east is the positive direction, plot student A's position (3.0 min, +300 m) above the time axis and student B's position (3.0 min, −450 m) below the time axis (Figure 1.17).



▲ **Figure 1.17**

(b) Convert time in minutes to time in seconds.

Then use the equation $\vec{v} = \dfrac{\Delta\vec{d}}{\Delta t}$ to find the velocity of each student.

$$\Delta t = 3.0 \text{ min} \times \frac{60 \text{ s}}{1 \text{ min}}$$

$$= 180 \text{ s}$$

$$\vec{v}_A = \frac{+300 \text{ m}}{180 \text{ s}}$$

$$= +1.7 \text{ m/s}$$

The sign is positive, so the direction is east.

$$\vec{v}_B = \frac{-450 \text{ m}}{180 \text{ s}}$$

$$= -2.5 \text{ m/s}$$

The sign is negative, so the direction is west.

Paraphrase

(b) Student A's velocity is 1.7 m/s [E] and student B's velocity is 2.5 m/s [W].

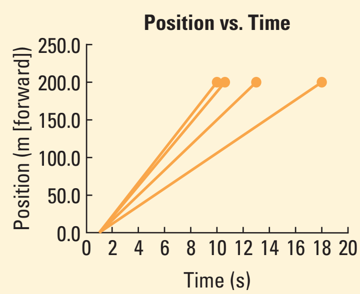
Practice Problems

1. A wildlife biologist measures how long it takes four animals to cover a displacement of 200 m [forward].
   (a) Graph the data from the table below.
   (b) Determine each animal's average velocity.

Animal	Time taken (s)
Elk	10.0
Coyote	10.4
Grizzly bear	18.0
Moose	12.9

Answers

1. (a)



(b) Elk: 20.0 m/s [forward]
Coyote: 19.2 m/s [forward]
Grizzly bear: 11.1 m/s [forward]
Moose: 15.5 m/s [forward]

So far, you have learned that the slope of a position–time graph represents velocity. By comparing the slopes of two graphs, you can determine which object is moving faster. From the slopes of the graphs in Figure 1.17, which student is moving faster? When you represent the motions of two objects on the same graph, you can also tell whether the objects are approaching or moving apart by checking if the lines are converging or diverging. An important event occurs at the point where the two lines intersect. Both objects have the same position, so the objects meet at this point.

Concept Check

Describe the shape of a graph showing the motion of two objects approaching each other.

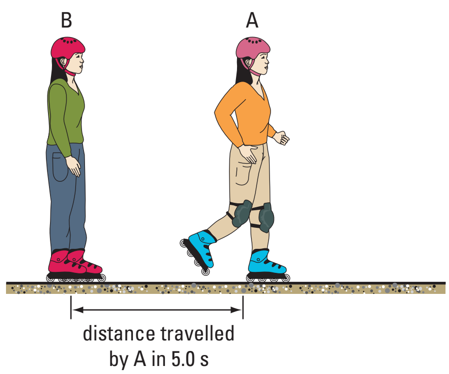
In the next example, two objects start at different times and have different speeds. You will graphically find their meeting point.

Example 1.3

Two rollerbladers, A and B, are having a race. B gives A a head start of 5.0 s (Figure 1.18). Each rollerblader moves with a constant velocity. Assume that the time taken to reach constant velocity is negligible. If A travels 100.0 m [right] in 20.0 s and B travels 112.5 m [right] in 15.0 s,

(a) graph the motions of both rollerbladers on the same graph.

(b) find the time, position, and displacement at which B catches up with A.



◄ **Figure 1.18**

Given

Choose right to be positive.

$\Delta\vec{d}_A = 100.0 \text{ m [right]} = +100.0 \text{ m}$

$\Delta t_A = 20.0 \text{ s}$

$\Delta\vec{d}_B = 112.5 \text{ m [right]} = +112.5 \text{ m}$

$\Delta t_B = 15.0$ s, started 5.0 s later

Required

(a) position-time graph

(b) time (Δt), position ($\vec{d}$), and displacement ($\Delta\vec{d}$) when B catches up with A

Practice Problems

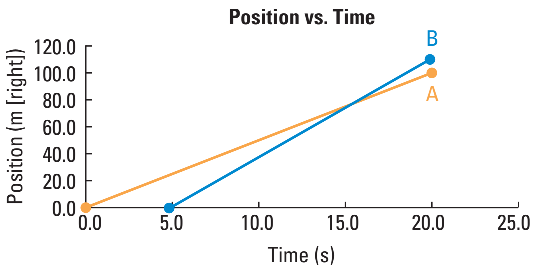
1. The two rollerbladers in Example 1.3 have a second race in which they each travel the original time and distance. In this race, they start at the same time, but B's initial position is 10.0 m left of A. Take the starting position of A as the reference.
   (a) Graph the motions of the rollerbladers.
   (b) Find the time, position, and B's displacement at which B catches up with A.

Answers

1. (b) $t = 4.0$ s
   $\vec{d} = 20.0$ m [right]
   $\Delta\vec{d} = 30.0$ m [right]

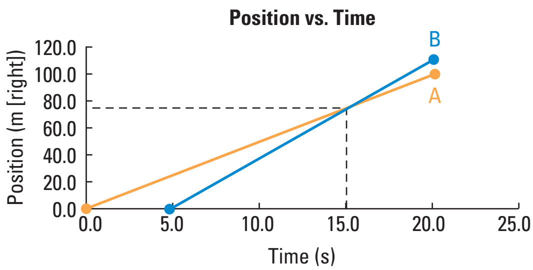
Analysis and Solution

(a) Assume that $t = 0.0$ s at the start of A's motion. Thus, the position-time graph of A's motion starts at the origin. A's final position is +100.0 m at 20.0 s.
The position-time graph for B's motion starts at 0.0 m and 5.0 s (because B started 5.0 s after A). B starts moving after 5.0 s for 15.0 s. Thus, at 20.0 s (5.0 s + 15.0 s), B's position is +112.5 m. Each rollerblader travels with a constant velocity, so the lines connecting their initial and final positions are straight (Figure 1.19(a)).



▲ **Figure 1.19(a)**

(b) On the graph in Figure 1.19(a), look for a point of intersection. At this point, both rollerbladers have the same final position. From the graph, you can see that this point occurs at $t = 15.0$ s. The corresponding position is +75.0 m (Figure 1.19(b)).



▲ **Figure 1.19(b)**

To find B's displacement, find the change in position: $\Delta\vec{d} = \vec{d}_f - \vec{d}_i$.
Both A and B started from the same position, $\vec{d}_i = 0$. Since they both have the same final position at the point of intersection, $\vec{d}_f = +75.0$ m.

$$\begin{aligned}\Delta\vec{d} &= +75.0 \text{ m} - 0.0 \text{ m} \\ &= +75.0 \text{ m}\end{aligned}$$

The answer is positive, so the direction is to the right.

Paraphrase

(b) B catches up with A 15.0 s after A started. B's position and displacement are 75.0 m [right] of the origin.

Example 1.4

From the graph in Example 1.3, find the velocities of the two rollerbladers.

Given

Choose right to be positive. At the point of intersection (Figure 1.19(b)),

$$\Delta\vec{d}_A = 75.0 \text{ m [right]} = +75.0 \text{ m}$$
$$\Delta t_A = 15.0 \text{ s}$$
$$\Delta\vec{d}_B = 75.0 \text{ m [right]} = +75.0 \text{ m}$$
$$\Delta t_B = 15.0 \text{ s} - 5.0 \text{ s} = 10.0 \text{ s}$$

Required

velocities of A and B ($\vec{v}_A$, $\vec{v}_B$)

Analysis and Solution

To find the velocity of each rollerblader, remember that the slope of a position–time graph is velocity. Because the motions are uniform, the slopes will be constant for each rollerblader.

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\vec{v}_A = \frac{+75.0 \text{ m} - 0.0 \text{ m}}{15.0 \text{ s} - 0.0 \text{ s}} = +5.0 \text{ m/s}$$

$$\vec{v}_B = \frac{+75.0 \text{ m} - 0.0 \text{ m}}{15.0 \text{ s} - 5.0 \text{ s}} = \frac{+75.0 \text{ m} - 0.0 \text{ m}}{+10.0 \text{ s}} = +7.5 \text{ m/s}$$

The answers are both positive, so the direction is to the right. You can see that, in order for B to cover the same distance as A, B must move faster because B started later.

Paraphrase

A's velocity is 5.0 m/s [right] and B's velocity is 7.5 m/s [right].

Practice Problems

1. Suppose rollerblader B gives A a head start of 5.0 s and takes 10.0 s to catch up with A at 100.0 m [right]. Determine the velocities of rollerbladers A and B.

Answers

1. A: 6.67 m/s [right]
   B: 10.0 m/s [right]

1-3 Inquiry Lab

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Car Activity

Question

What are the speeds of two different toy cars?
If one car is released 3.0 s after the other, where will they meet?

Variables

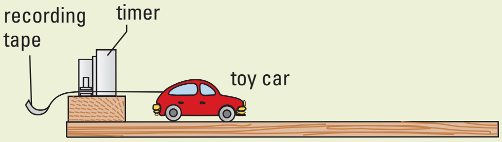
Identify the manipulated, responding, and controlled variables.

Materials and Equipment

two battery-operated toy cars	
ticker tape	ruler
carbon disk	graph paper
spark timer (60 Hz)	masking tape

Procedure

1. On a flat surface, such as the floor or lab bench, mark the initial starting position of car 1 with masking tape.
2. Using masking tape, attach 1.0 m of ticker tape to the end of car 1.
3. Thread the ticker tape through the spark timer (Figure 1.20).
4. Turn the car on.
5. Turn the spark timer on as you release the car from its initial position.
6. Observe the path of car 1 until the ticker tape is used up. Label the ticker tape "car 1."
7. Repeat steps 2–6 for car 2.



▲ **Figure 1.20**

Analysis

1. Draw a line through the first dot on each ticker tape and label it $t = 0$ s.
2. Depending on the calibration of your ticker timer, count from the starting position, and place a mark after a fixed number of dots, e.g., 6, 12, 18, 24, etc. Label each mark t_1, t_2, etc. On a 60-Hz timer, every sixth dot represents 0.10 s.
3. Measure the distance from $t = 0$ to t_1, $t = 0$ to t_2, $t = 0$ to t_3, etc. Record the data in a position–time table.
4. Using an appropriate scale, graph each set of data for each toy car, separately.
5. Determine the slope of the line of best fit for each graph. See pages 872–873 for explicit instruction on how to draw a line of best fit.
6. What is the speed of each toy car?
7. How do the speeds of car 1 and car 2 compare?
8. Assuming uniform motion, how far would car 1 travel in 15 s?
9. Assuming uniform motion, how long would it take car 2 to travel 30 m?
10. Imagine that you release the faster car 3.0 s after the slower car. Graphically determine the position where the two cars meet. Assume uniform motion.

***e*LAB**

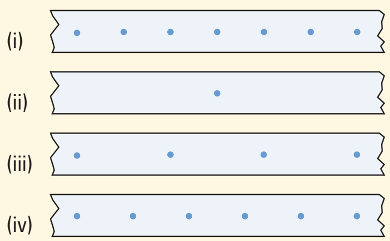
For a probeware activity, go to www.pearsoned.ca/school/physicssource.

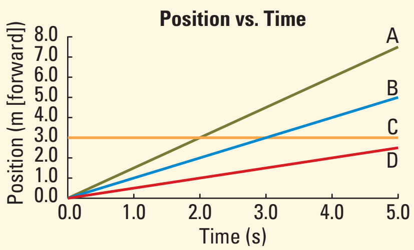
In summary, you can see how a position-time graph helps you visualize the event you are analyzing. Calculating the slope of a position-time graph provides new information about the motion, namely, the object's velocity. In the next sections, you will expand on your graphing knowledge by analyzing motion using a velocity-time graph.

1.2 Check and Reflect

Knowledge

1. For an object at rest, what quantities of motion remain the same over equal time intervals?
2. For an object travelling at a constant velocity, what quantity of motion remains the same over equal time intervals?
3. Match each ticker tape below with the correct position-time graph.

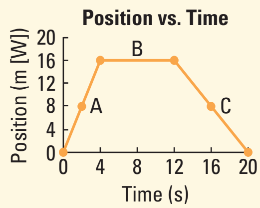




4. Two friends start walking on a football field in the same direction. Person A walks twice as fast as person B. However, person B has a head start of 20.0 m. If person A walks at 3.0 m/s, find the distance between the two friends after walking for 20.0 s and determine who is ahead at this time. Sketch a position-time graph for both people.
5. A camper kayaks 16 km [E] from a camping site, stops, and then paddles 23 km [W]. What is the camper's final position with respect to the campsite?
6. Sketch a position-time graph for a bear starting 1.2 m from a reference point, walking slowly away at constant velocity for 3.0 s, stopping for 5.0 s, backing up at half the speed for 2.0 s, and finally stopping.
7. Sketch a position-time graph for a student
   (a) walking east to school with a constant velocity
   (b) stopping at the school, which is 5 km east of home
   (c) cycling home with a constant velocity

Applications

8. Two children on racing bikes start from the same reference point. Child A travels 5.0 m/s [right] and child B travels 4.5 m/s [right]. How much farther from the point of origin is child A than child B after 5.0 s?
9. Insect A moves 5.0 m/min and insect B moves 9.0 cm/s. Determine which insect is ahead and by how much after 3.0 min. Assume both insects are moving in the same direction.
10. Describe the motion in each lettered stage for the object depicted by the position-time graph below.



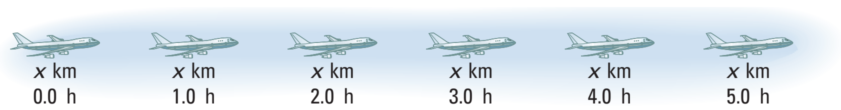
11. A mosquito flies toward you with a velocity of 2.4 km/h [E]. If a distance of 35.0 m separates you and the mosquito initially, at what point (distance and time) will the mosquito hit your sunglasses if you are travelling toward the mosquito with a speed of 2.0 m/s and the mosquito is travelling in a straight path?
12. Spotting a friend 5.0 m directly in front of you, walking 2.0 m/s [N], you start walking 2.25 m/s [N] to catch up. How long will it take for you to intercept your friend and what will be your displacement?
13. Two vehicles, separated by a distance of 450 m, travel in opposite directions toward a traffic light. When will the vehicles pass one another if vehicle A is travelling 35 km/h and is 300 m [E] of the traffic light while vehicle B is travelling 40 km/h? When will each vehicle pass the traffic light, assuming the light remains green the entire time?

eTEST

To check your understanding of uniform motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

1.3 Velocity-time Graphs: Uniform and Non-uniform Motion

Recently installed video screens in aircraft provide passengers with information about the aircraft's velocity during the flight (Figure 1.21).



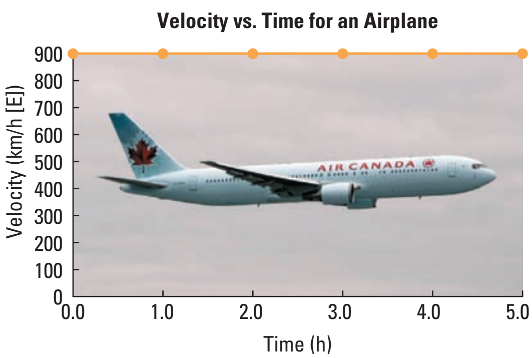
▲ **Figure 1.22** A plane flies at a constant speed, so the distances within each time interval are equal. Break the plane's motion into a series of snapshots. Record your data in a data table and then graph it.

▲ **Figure 1.21** Video screens are an example of an application of velocity-time graphs.

Figure 1.22 shows the data of the plane's path. Like position-time graphs, velocity-time graphs provide useful information about the motion of an object. The shape of the velocity-time graph reveals whether the object is at rest, moving at constant speed, speeding up, or slowing down. Suppose an airplane has a cruising altitude of 10 600 m and travels at a constant velocity of 900 km/h [E] for 5.0 h. Table 1.3 shows the velocity-time data for the airplane. If you graph the data, you can determine the relationship between the two variables, velocity and time (Figure 1.23).

▼ **Table 1.3**

Time (h)	Velocity (km/h) [E]
0.0	900
1.0	900
2.0	900
3.0	900
4.0	900
5.0	900



▲ **Figure 1.23** A velocity-time graph for an airflight

Designating east as the positive direction, the slope of the velocity-time graph is:

$$\begin{aligned}
\text{slope} &= \frac{\text{rise}}{\text{run}} \\
&= \frac{\Delta \vec{v}}{\Delta t} \\
&= \frac{\vec{v}_\text{f} - \vec{v}_\text{i}}{t_\text{f} - t_\text{i}} \\
&= \frac{+900\,\frac{\text{km}}{\text{h}} - \left(+900\,\frac{\text{km}}{\text{h}}\right)}{5.0\text{ h} - 1.0\text{ h}} \\
&= 0\text{ km/h}^2
\end{aligned}$$

From the graph in Figure 1.23, there is no change in the plane's velocity, so the slope of the velocity-time graph is zero.

Notice the units of the slope of the velocity-time graph: km/h^2. These units are units of *acceleration*. Because the plane is moving at a constant velocity, its acceleration is zero.

In general, you can recognize acceleration values by their units, which are always distance divided by time squared. In physics, the standard units for acceleration are metres per second per second, which is generally abbreviated to m/s^2 (read metres per second squared).

eTECH

Determine the velocity of an object based on the shape of its position-time graph. Go to www.pearsoned.ca/school/physicssource.

Concept Check

(a) What does the slope of a position-time graph represent?
(b) What does the slope of a velocity-time graph represent?

Non-uniform Motion

Although objects may experience constant velocity over short time intervals, even a car operating on cruise control has fluctuations in speed or direction (Figure 1.24). How can you describe and illustrate a change of velocity using the concepts of kinematics?

Recall from section 1.2 that an object moving at a constant velocity is undergoing uniform motion. But is uniform motion the only type of motion? Perform the next QuickLab to find out.

◀ **Figure 1.24** Consider the kinds of changes in velocity this car experiences during the trip.

1-4 QuickLab

Match a Graph

Problem

What type of motion does each graph describe?

Materials

LM 1-1 (provided by your teacher)
ruler
motion sensor
masking tape

Procedure

1. Study the different position-time graphs on LM 1-1. With a partner, decide what type of motion each graph illustrates.
2. Set up the motion sensor to plot position vs. time.
3. Label a starting position with masking tape approximately 1 m in front of the motion sensor. Move away from the motion sensor in such a way that the graph of the motion captured approximates the one on the LM.
4. Switch roles with your partner and repeat steps 1–3.
5. Print out the graphs from your experiment. For each graph, construct a table of values for position and time.

Questions

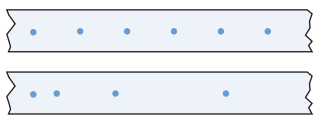
1. Describe your motion when a horizontal line was being produced on the position–time graph.
2. What relationship exists between the type of motion and change in position?
3. Suggest two different ways in which you could classify the motion described by the four graphs.
4. What would the graph look like if you moved away from and then back toward the motion sensor?
5. What happens to the graph when you move away from your initial position and then move back toward and then beyond your initial position?

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Concept Check

Which ticker tape in Figure 1.25 represents accelerated motion? Explain.

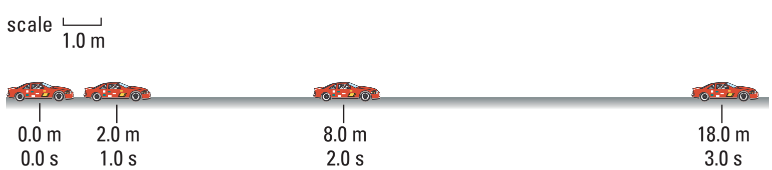


▲ **Figure 1.25**

▲ **Figure 1.26** A drag racer accelerates from rest.

Consider an object, such as a drag racer (Figure 1.26), starting from rest and reaching a constant velocity over a time interval (Figure 1.27). During this time interval, the vehicle has to change its velocity from a value of zero to a final non-zero value. An object whose velocity changes (increases or decreases) over a time interval is undergoing **acceleration**, represented by the variable $\vec{a}$. Acceleration is a **vector quantity**. It is also called **non-uniform motion** because the object's speed or direction is changing.

acceleration: a vector quantity representing the change in velocity (magnitude or direction) per unit time



◀ **Figure 1.27** This sequence illustrates a car undergoing non-uniform motion.

PHYSICS INSIGHT

An object is accelerating if it is speeding up, slowing down, or changing direction.

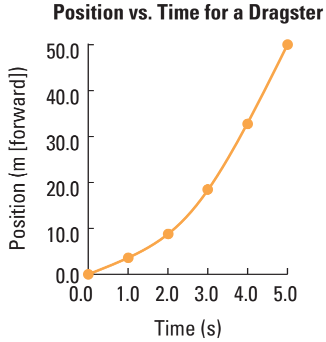
The following scenario illustrates acceleration.

A drag race is a 402-m (quarter-mile) contest between two vehicles. Starting from rest, the vehicles leave the starting line at the same time, and the first vehicle to cross the finish line is the winner. A fan records the position of her favourite vehicle during the drag race. Her results are recorded in Table 1.4.

The position-time graph for this data is shown in Figure 1.28. From the graph, note that the object is speeding up because the displacement between data points increases for each successive time interval. Which ticker tape in Figure 1.25 matches the graph in Figure 1.28?

▼ **Table 1.4**

Time (s)	Position (m [forward])
0.0	0.0
1.0	2.0
2.0	8.0
3.0	18.0
4.0	32.0
5.0	50.0



◀ **Figure 1.28** What does the slope of the graph indicate about the speed of the car?

Instantaneous Velocity

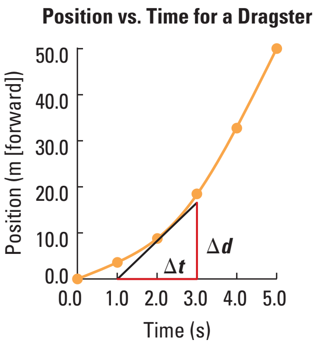
PHYSICS INSIGHT

When you calculate the slope of a line or curve at a single point, you are finding an instantaneous value. When you calculate the slope between two points, you are finding an average value.

tangent: a straight line that touches a curved-line graph at only one point

Instantaneous velocity is the moment-to-moment measure of an object's velocity. Imagine recording the speed of your car once every second while driving north. These data form a series of instantaneous velocities that describe your trip in detail.

Earlier in this section, you learned that determining the velocity of an object from a position-time graph requires calculating the slope of the position-time graph. But how can you obtain the slope of a curve? Remember that each point on the curve indicates the position of the object (in this case, the dragster) at an instant in time. To determine the velocity of an object at any instant, physicists use tangents. A **tangent** is a straight line that touches a curve at only one point (Figure 1.29(a)). Each tangent on a curve has a unique slope, which represents the velocity at that instant. In order for the object to be at that position, at that time, it must have an *instantaneous velocity* equal to the slope of the tangent at that point. Determining the slopes of the tangents at different points on a position-time curve gives the instantaneous velocities at different times. Consider forward to be the positive direction.



▲ **Figure 1.29(a)**

The slope of the tangent at 2.0 s is

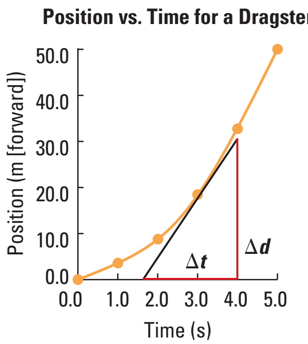
$$\text{slope} = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{+14.0\ \text{m} - 0.0\ \text{m}}{3.0\ \text{s} - 1.0\ \text{s}}$$

$$= \frac{+14.0\ \text{m}}{2.0\ \text{s}}$$

$$= +7.0\ \text{m/s}$$

The sign is positive, so at 2.0 s, the velocity of the dragster is 7.0 m/s [forward].



▲ **Figure 1.29(b)**

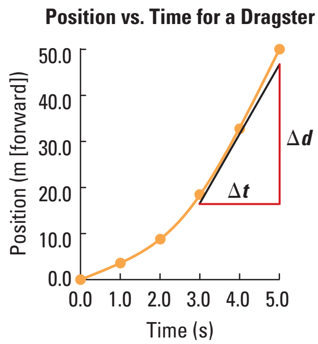
The slope of the tangent at 3.0 s is

$$\text{slope} = \frac{+30.0\ \text{m} - 0.0\ \text{m}}{4.0\ \text{s} - 1.75\ \text{s}}$$

$$= \frac{+30.0\ \text{m}}{2.25\ \text{s}}$$

$$= +13\ \text{m/s}$$

At 3.0 s, the velocity of the dragster is 13 m/s [forward].



▲ **Figure 1.29(c)**

The slope of the tangent at 4.0 s is

$$\text{slope} = \frac{+47.0\ \text{m} - (+15.0\ \text{m})}{5.0\ \text{s} - 3.0\ \text{s}}$$

$$= \frac{+32.0\ \text{m}}{2.0\ \text{s}}$$

$$= +16\ \text{m/s}$$

At 4.0 s, the velocity of the dragster is 16 m/s [forward].

Using Slopes of Position-time Graphs to Draw Velocity-time Graphs

You can now create a new table using the slopes of the position-time graphs in Figures 1.29(a), (b), and (c). See Table 1.5. Remember that the slope of a position-time graph is velocity. These slope values are actually *instantaneous velocities* at the given times. You can use these three velocities to draw a velocity-time graph (Figure 1.30). The resulting velocity-time graph is a straight line that goes through the origin when extended. This means that the dragster has started from rest (0 velocity). The graph has a positive slope. To find the meaning of slope, check the units of the slope of a velocity-time graph. They are (m/s)/s, which simplify to m/s^2. These units are the units of acceleration. Since the velocity-time graph in this example is a straight line with non-zero slope, the acceleration of the object is constant, so the object must be undergoing **uniformly accelerated motion.**

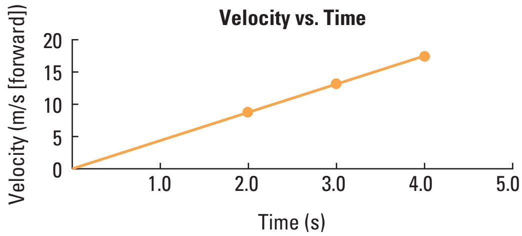
info **BIT**

When jet fighters come in to land on an aircraft carrier, they stop so quickly that pilots sometimes lose consciousness for a few seconds. The same thing can happen when a pilot ejects from an aircraft, due to enormous acceleration.

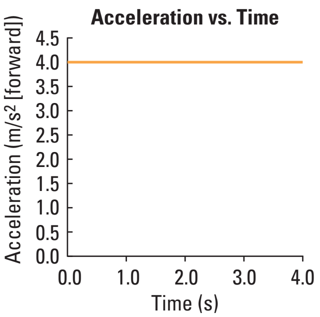
uniformly accelerated motion: constant change in velocity per unit time

▼ **Table 1.5**

Time (s)	Velocity (m/s [forward])
2.0	7.0
3.0	13
4.0	16



▲ **Figure 1.30** This velocity-time graph represents an object undergoing uniformly accelerated motion.



▲ **Figure 1.31** An acceleration-time graph for an object undergoing uniformly accelerated motion is a straight line with zero slope.

Just as the slope of a position-time graph reveals the rate at which position changes (velocity), the slope of a velocity-time graph reveals the rate at which velocity changes (acceleration). Calculate the slope of the line in Figure 1.30 as follows, designating forward as positive:

$$\text{slope} = \frac{\text{rise}}{\text{run}}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{+10\ \text{m/s} - (+4\ \text{m/s})}{2.5\ \text{s} - 1.0\ \text{s}}$$

$$= +4\ \text{m/s}^2$$

The answer is positive, so the car is accelerating at 4 m/s^2 [forward]. The resulting acceleration-time graph is shown in Figure 1.31. You know that the velocity-time graph for an object undergoing uniform motion is a horizontal line (with zero slope, as in Figure 1.23). Similarly, a horizontal line on an acceleration-time graph indicates uniform acceleration.

PHYSICS INSIGHT

If the acceleration-time graph has a non-zero slope, the acceleration is changing (is non-uniform). The slope of an acceleration-time graph is called *jerk*, with units m/s^3.

Concept Check

If the position-time graph for an object undergoing positive acceleration is a parabola, such as the one in Figure 1.28, what is the shape of the position-time graph for an object undergoing negative acceleration? What would a ticker tape of the motion of an object that is slowing down look like?

After driving your all-terrain vehicle (ATV, Figure 1.32) through a field, you see a wide river just ahead, so you quickly bring the vehicle to a complete stop. Notice in Figure 1.33 that, as your ATV slows down, the displacement in each time interval decreases.

◄ **Figure 1.32** ATVs can undergo a wide variety of motions.

▲ **Figure 1.33** This ATV is undergoing non-uniform motion. It is accelerating, in this case, slowing down.

Example 1.5 shows the calculations and resulting velocity-time graph for an object that is slowing down uniformly.

Example 1.5

The position-time data for an ATV approaching a river are given in Table 1.6. Using these data,
(a) draw a position-time graph
(b) draw a velocity-time graph
(c) calculate acceleration

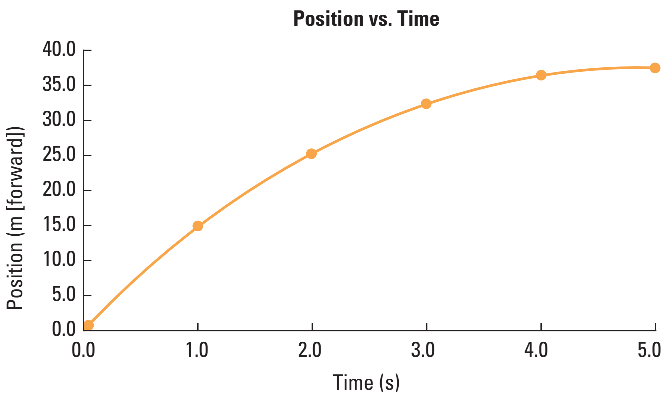
▼ **Table 1.6**

Time (s)	Position (m [forward])
0.0	0.0
1.0	13.5
2.0	24.0
3.0	31.5
4.0	36.0
5.0	37.5

Analysis and Solution

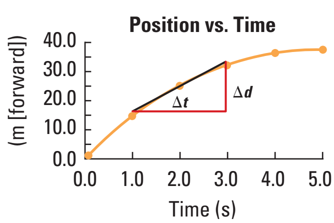
Designate the forward direction as positive.

(a) For the position-time graph, plot the data in Table 1.6 (Figure 1.34).



▲ **Figure 1.34**

(b) Since the position-time graph is non-linear, find the slope of the tangent at 2.0 s, 3.0 s, and 5.0 s (Figures 1.35(a), (b), and (c)).



▲ **Figure 1.35(a)**

$$\text{slope} = \frac{\Delta \vec{d}}{\Delta t}$$

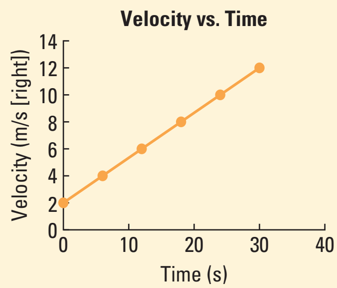
$$= \frac{+32.5 \text{ m} - 15.5 \text{ m}}{3.0 \text{ s} - 1.0 \text{ s}}$$

$$= \frac{+17.0 \text{ m}}{2.0 \text{ s}}$$

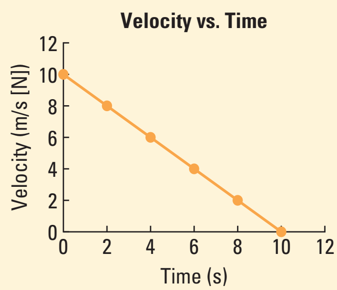
$$= +8.5 \text{ m/s}$$

Practice Problems

1. Draw a position-time graph from the velocity-time graph given below.

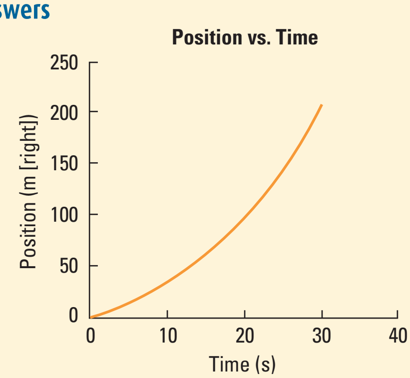


2. Calculate the acceleration using the graph below.

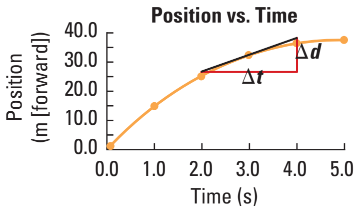


Answers

1.



2. -1.0 m/s^2 [N]



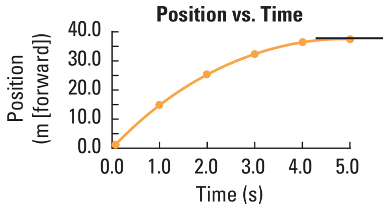
▲ **Figure 1.35(b)**

$$\text{slope} = \frac{\Delta \vec{d}}{\Delta t}$$

$$= \frac{+37.0\text{ m} - (+26.0\text{ m})}{4.0\text{ s} - 2.0\text{ s}}$$

$$= \frac{+11.0\text{ m}}{2.0\text{ s}}$$

$$= +5.5\text{ m/s}$$



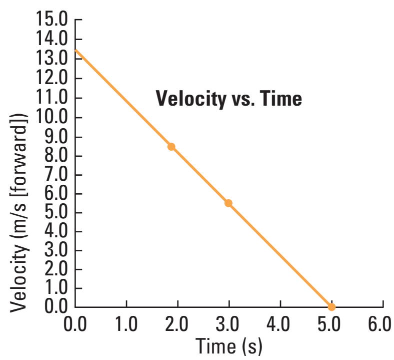
This tangent is a horizontal line, so its slope is zero.

▲ **Figure 1.35(c)**

The slopes of the tangents give the instantaneous velocities (Table 1.7). Positive signs mean that the direction is forward. Plot the data on a velocity-time graph (Figure 1.36).

▼ **Table 1.7**

Time (s)	Velocity (m/s [forward])
2.0	8.5
3.0	5.5
5.0	0



▲ **Figure 1.36**

(c) Find acceleration by calculating the slope of the velocity-time graph.

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$= \frac{0.0\text{ m/s} - (+8.5\text{ m/s})}{5.0\text{ s} - 2.0\text{ s}}$$

$$= -2.8\text{ m/s}^2$$

The acceleration of the ATV is -2.8 m/s^2. Because the forward direction was designated as positive, the negative sign means that the direction of acceleration is backward.

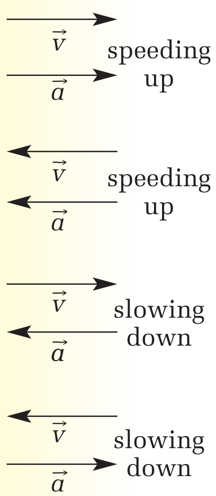
Negative Acceleration Does Not Necessarily Mean Slowing Down

In Example 1.5, the value for acceleration is negative. What is the meaning of negative acceleration? When interpreting the sign of acceleration, you need to compare it to the sign of velocity. For example, for the drag racer that is speeding up, the direction of its velocity is the same as the direction of its acceleration (see the calculation of the slope of the velocity-time graph for Figure 1.30). *When the directions (signs) of velocity and acceleration are the same (positive or negative), the object is speeding up.*

e MATH

For an alternative method to create a velocity-time graph from the position-time data points, visit www.pearsoned.ca/school/physicssource.

PHYSICS INSIGHT



For the ATV in Example 1.5, the direction of its velocity is opposite to the direction of its acceleration, so it is slowing down. *When velocity and acceleration have opposite directions (signs), the object slows down.*

Concept Check

(a) Think of two more examples of objects not mentioned in this text that are speeding up and slowing down. In each case, indicate the signs or directions of velocity and acceleration.

(b) Under what circumstances can an object have a negative acceleration and be speeding up?

(c) You are given a position-time graph that is a curve. How can you use the slope of the tangent to determine whether the object represented in the graph is speeding up or slowing down? (Hint: How does the slope of the tangent change as you move along the position-time curve?)

THEN, NOW, AND FUTURE — Biomechanics and the Role of the Crash Test Dummy

Understanding how biological systems move is a branch of physics known as biomechanics. For automobile manufacturers, understanding how the human body moves during a car accident is very important. To study, collect, and analyze data on how the human body moves during a vehicular collision requires a test subject.

Human cadavers were the first test subjects used. While live human testing was valuable, it was limited in its scope due to the physical discomfort required and injury potential for some of the tests. Despite the production of reliable applicable data, most automobile manufacturers discontinued live animal testing in 1993 for moral and ethical reasons.

Clearly, a different type of test subject needed to be designed and built. It came in the form of the now recognizable crash test dummy.

Sam W. Alderson created "Sierra Sam" in 1949 to test aircraft ejection seats and pilot restraint harnesses. Then came the VIP-50 series and Sierra Stan in the 1950s. Engineers combined the best features of these different models and debuted Hybrid I in 1971. Hybrid I was known as the "50th percentile male" dummy (meaning approximately 50% of men are larger and 50% of men are smaller), with a height of 168 cm and a mass of 77 kg. A year later, Hybrid II, with improved shoulder, spine, and knee responses, was produced to test lap and shoulder belts. Still crude, their use was limited, leading to the advent of the Hybrid III family of crash test dummies that include a 50th percentile male, a 50th percentile female, and two child dummies. This family of crash test dummies is designed to measure spine and rib acceleration, and demonstrate neck movement in rear-end collisions.

Equipped with a more human-like spine and pelvis, THOR (Figure 1.37) is the successor of Hybrid III. Its face contains a number of sensors for facial impact analysis. Since front and side air bags have reduced upper body injury, lower extremity injury has become more prevalent. Therefore, THOR is built with an Achilles tendon to better mimic the side-to-side, up-and-down, and rotational movements of the ankle.

Even with sophisticated crash test dummies, plastic and steel can only approximate how the human body will move. The study of soft tissue injury can only be accomplished with real-life subjects. Therefore, the future of crash testing will be in creating detailed computer models of human systems. Even though it is slow and cumbersome for full body simulations, the computer has the advantage of repeatability and lower cost. The programmer has the ability to control every variable and repeat each and any event.

1. Why are crash test dummies used?
2. What are some of the advantages of THOR over his previous prototypes?
3. Will crash test dummies become obsolete? Explain.

▲ **Figure 1.37** THOR

1.3 Check and Reflect

Applications

1. A sprinter in a championship race accelerates to his top speed in a short time. The velocity-time data for part of the race are given in the table below. Use the data to find the

(a) average acceleration from 0.00 s to 0.50 s

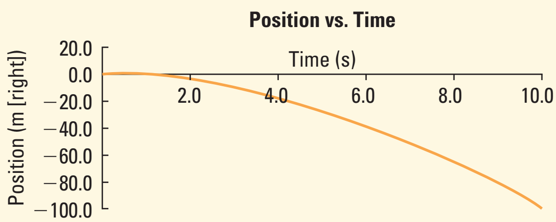
(b) average acceleration from 0.50 s to 3.00 s

(c) average acceleration from 5.00 s to 6.00 s

(d) Describe what was happening to the acceleration and velocity over 6.00 s.

Time (s)	Velocity (m/s [forward])
0.00	0.00
0.12	0.00
0.14	0.00
0.50	2.80
1.00	5.00
2.00	8.00
3.00	9.80
4.00	10.80
5.00	11.30
6.00	11.60
7.00	11.70
8.00	11.80
9.00	11.90
9.83	11.95
9.93	11.97

2. Describe the motion of the object as illustrated in the graph below.



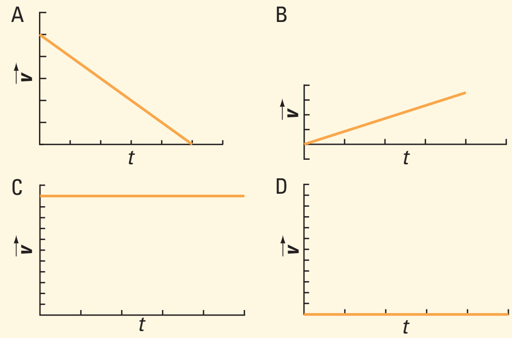
3. Match each velocity-time graph below with the correct statement.

(i) negative acceleration

(ii) positive acceleration

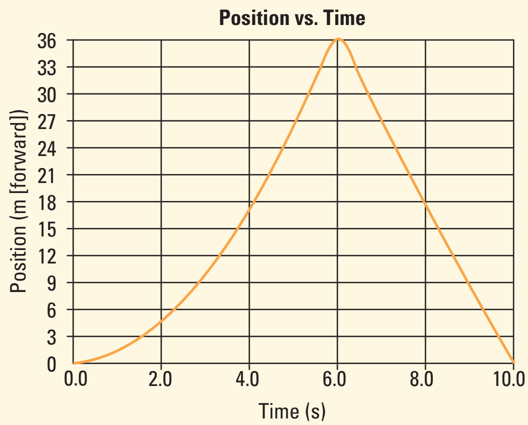
(iii) moving with zero acceleration

(iv) stationary object



Extensions

4. In your notebook, complete the velocity-time data table for the graph below.



Time (s)	Velocity (m/s [forward])
2.0	
4.0	
6.0	
8.0	

e TEST

To check your understanding of uniformly accelerated motion, follow the eTest links at www.pearsoned.ca/school/physicssource.

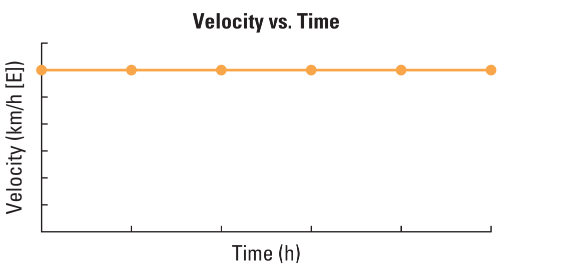
1.4 Analyzing Velocity-time Graphs

When a plane flies across Alberta with constant speed and direction, it is said to be undergoing *uniform motion* (Figure 1.38(a)). If you were sitting in the plane, you would experience a smooth ride. An all-terrain vehicle (ATV) bouncing and careening along a rough trail is constantly changing speed and direction in order to stay on the road. A ride in the ATV illustrates *non-uniform motion*, or acceleration (Figure 1.39(a)).

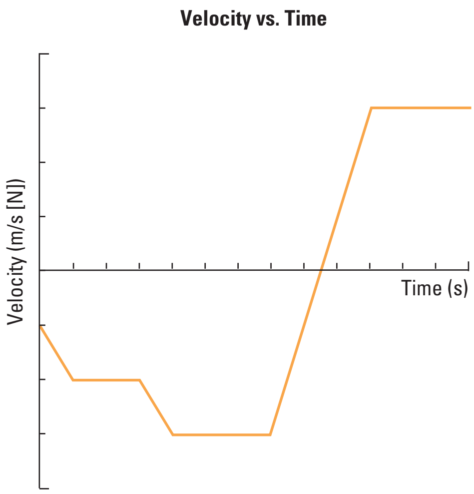
You can distinguish between uniform and non-uniform motion by simple observation and gathering data from your observations (see Figures 1.38(b) and 1.39(b)). There are several ways to interpret the data. One way is to analyze graphs by determining their slopes to obtain further information about an object's motion, as you did in section 1.3. In this section, you will develop this method further and learn another method of graphical analysis: how to find the area under a graph. First review the information you can obtain from the slopes of position-time and velocity-time graphs.

▲ **Figure 1.38(a)** Uniform motion

▲ **Figure 1.39(a)** Non-uniform motion



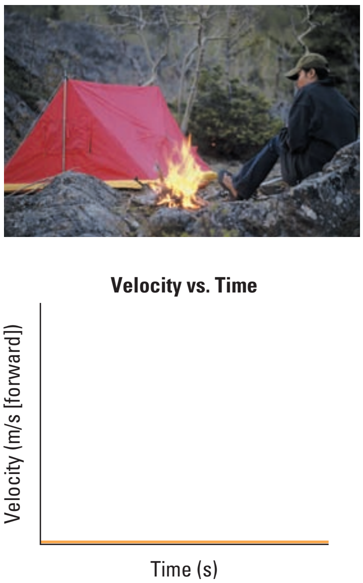
▲ **Figure 1.38(b)** A graph representing uniform motion



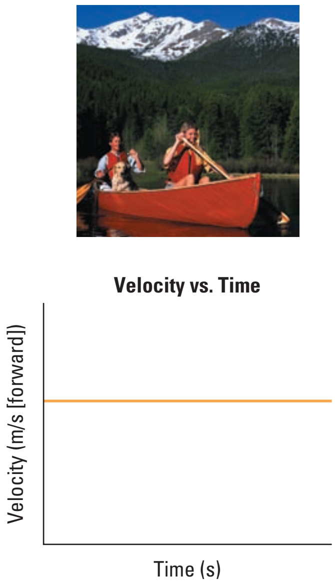
▲ **Figure 1.39(b)** A graph representing non-uniform motion

Slopes of Graphs Reveal How Objects Move

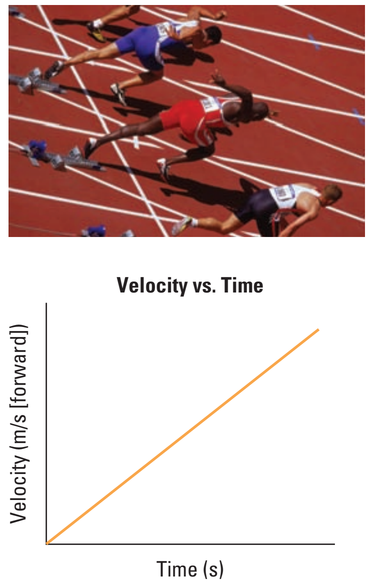
Consider the three photos and velocity-time graphs in Figure 1.40. You can interpret each graph by reading values from it. To gain new information, you must analyze the graph by calculating its slope. The slope describes the object's motion.



▲ **Figure 1.40(a)** A velocity-time graph for an object at rest



▲ **Figure 1.40(b)** A velocity-time graph for an object undergoing uniform motion



▲ **Figure 1.40(c)** A velocity-time graph for an object undergoing uniformly accelerated motion

Concept Check

1. Sketch position-time graphs for the following:
   (a) two possibilities for stopped motion
   (b) two possibilities for uniform motion
   (c) four possibilities for uniformly accelerated motion

   Describe each graph in terms of direction of travel and whether the object is speeding up or slowing down.
2. Sketch the corresponding velocity-time graph for each position-time graph in question 1. For each graph, describe how you determined the graph's shape.

By analyzing the units for the slope of a velocity-time graph, m/s^2, you know from section 1.3 that the slope of a velocity-time graph represents the acceleration of the object.

Concept Check

Sketch all the types of acceleration-time graphs you have encountered thus far. Describe the kind of motion that each graph represents.

1-5 Design a Lab

Tortoise or Hare?

The Question

In your class, who has the fastest acceleration and the fastest average speed in the 50-m dash?

Design and Conduct Your Investigation

Make a list of variables that you think are likely to influence the acceleration of each participant. For each variable on your list, write a hypothesis that predicts how changes in that variable will affect the participants' acceleration. Write a procedure for an investigation that will test the effect of one of these variables on acceleration. Clearly outline all the steps that you will follow to complete your investigation. Identify the responding and manipulated variables. List all the materials and equipment you will need, as well as all safety precautions. Compare your experimental design and procedure with those of your classmates. Identify any strengths and weaknesses. With your teacher's approval, conduct your investigation. State any problems or questions that you found during your investigation or analysis that would need additional investigation to answer.

The Area Under a Velocity-time Graph Represents Displacement

Occasionally, due to a medical or other emergency, a pilot must turn the aircraft and land at the same or alternate airport. Consider the graph for the uniform motion of a plane travelling east at 300 km/h for 2.0 h only to turn back west for 0.5 h to make an emergency landing (Figure 1.41). What is the plane's displacement for this time interval?

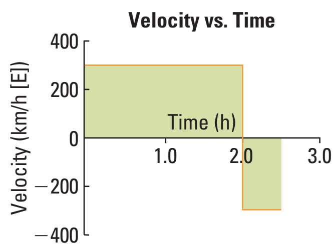
Unit analysis indicates that the area under a velocity-time graph equals displacement. To calculate displacement using a velocity-time graph, look at the units on the axes. To end up with a unit of displacement (km) from the units km/h and h, you need to multiply:

$$\frac{\text{km}}{\cancel{\text{h}}} \times \cancel{\text{h}} = \text{km}$$

The shapes in Figure 1.41 are rectangles, so the area under the velocity-time graph is $l \times w$ (length times width). In this case, find the sum of the areas above and below the time axis. Consider east to be positive. For eastward displacement, the area is *above* the time axis, so it is positive. For westward displacement, the area is *below* the time axis, so it is negative.

For eastward displacement (above the time axis),

$$\begin{aligned} \text{area} = \Delta\vec{d} &= \vec{v}\Delta t \\ &= \left(+300\ \frac{\text{km}}{\cancel{\text{h}}}\right)(2.0\ \cancel{\text{h}}) \\ &= +600\ \text{km} \end{aligned}$$



▲ **Figure 1.41** To calculate net displacement, add the areas above and below the time axis.

For westward displacement (below the time axis),

$$\Delta\vec{d} = \vec{v}\Delta t$$
$$= \left(-300\ \frac{\text{km}}{\not{\text{h}}}\right)(0.5\ \not{\text{h}})$$
$$= -150\ \text{km}$$

To find the plane's net displacement, add the two areas.

$$\text{area} = \Delta\vec{d} = +600\ \text{km} + (-150\ \text{km})$$
$$= +600\ \text{km} - 150\ \text{km}$$
$$= +450\ \text{km}$$
$$= 5 \times 10^2\ \text{km [E]}$$

Because the net area is positive, the plane's displacement is 5×10^2 km [E].

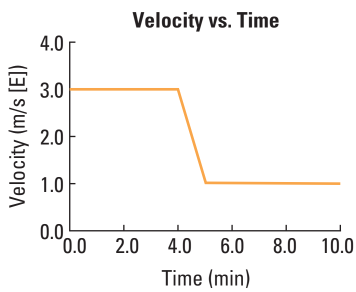
Unlike position-time graphs, where you can only calculate the slope to determine velocity, you can use velocity-time graphs to determine both acceleration and displacement, as in the next example.

PHYSICS INSIGHT

Note that the answer has one significant digit because there is only one significant digit in 0.5 h.

Example 1.6

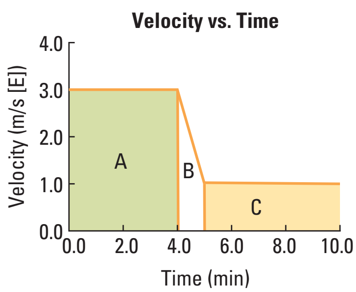
From the graph in Figure 1.42, calculate
(a) displacement
(b) acceleration



▲ **Figure 1.42**

Analysis and Solution

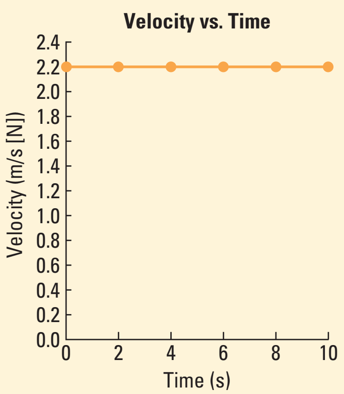
(a) For displacement, find the sum of the areas under the velocity-time graph (Figure 1.43). Designate east (above the time axis) as the positive direction. Convert minutes to seconds.



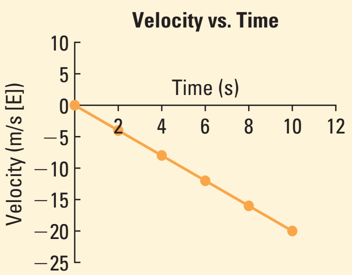
▲ **Figure 1.43**

Practice Problems

1. Calculate the displacement and acceleration from the graph.



2. Use the graph below to determine the displacement of the object.



Answers

1. 22 m [N], 0 m/s^2 [N]
2. -1.0×10^2 m [E] or 1.0×10^2 m [W]

Region A:

$$\Delta\vec{d}_A = \vec{v}\Delta t$$

$$= \left(+3.0\ \frac{\text{m}}{\text{s}}\right)\left(4.0\ \cancel{\text{min}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right)$$

$$= \left(+3.0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(240\ \cancel{\text{s}})$$

$$= +720\ \text{m}$$

Region B:

$$\Delta\vec{d}_B = \frac{1}{2}(5.0\ \text{min} - 4.0\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)\left(+3.0\ \frac{\text{m}}{\text{s}} - 1.0\ \frac{\text{m}}{\text{s}}\right) + \left(+1.0\ \frac{\text{m}}{\text{s}}\right)(5.0\ \text{min} - 4.0\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \frac{1}{2}(1.0\ \cancel{\text{min}})\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)\left(+2.0\ \frac{\text{m}}{\cancel{\text{s}}}\right) + \left(+1.0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(1.0\ \cancel{\text{min}})\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)$$

$$= \frac{1}{2}(120\ \text{m}) + 60\ \text{m}$$

$$= +120\ \text{m}$$

Region C:

$$\Delta\vec{d}_C = \left(+1.0\ \frac{\text{m}}{\text{s}}\right)(10.0\ \text{min} - 5.0\ \text{min})\left(\frac{60\ \text{s}}{1\ \text{min}}\right)$$

$$= \left(+1.0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(5.0\ \cancel{\text{min}})\left(\frac{60\ \cancel{\text{s}}}{1\ \cancel{\text{min}}}\right)$$

$$= +300\ \text{m}$$

$$\Delta\vec{d} = \Delta\vec{d}_A + \Delta\vec{d}_B + \Delta\vec{d}_C$$

$$= +720\ \text{m} + 120\ \text{m} + 300\ \text{m}$$

$$= +1140\ \text{m}$$

$$= +1.1 \times 10^3\ \text{m}$$

The answer is positive, so the direction of the displacement is east.

(b) For acceleration, find the slope of each section of the graph.

In region A:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{+3.0\ \frac{\text{m}}{\text{s}} - \left(+3.0\ \frac{\text{m}}{\text{s}}\right)}{240\ \text{s}}$$

$= 0.0\ \text{m/s}^2$ This answer makes sense because the velocity-time graph is a horizontal line.

In region B:

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

$$= \frac{+1.0\ \frac{\text{m}}{\text{s}} - \left(+3.0\ \frac{\text{m}}{\text{s}}\right)}{60\ \text{s}}$$

$= -0.033\ \text{m/s}^2$ or $0.033\ \text{m/s}^2$[W]

PHYSICS INSIGHT

When calculating total displacement from a velocity-time graph, remember to keep track of whether the area is positive or negative.

PHYSICS INSIGHT

Check your answer by looking at the units. Do the units reflect the answer that you are asked to find?

Since the third part of the graph is also a horizontal line, its slope is also zero.

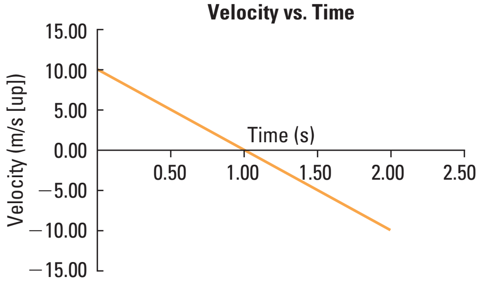
Paraphrase

(a) The displacement is 1.1×10^3 m [E].

(b) The acceleration is zero in regions A and C and 0.033 m/s² [W] in region B.

Concept Check

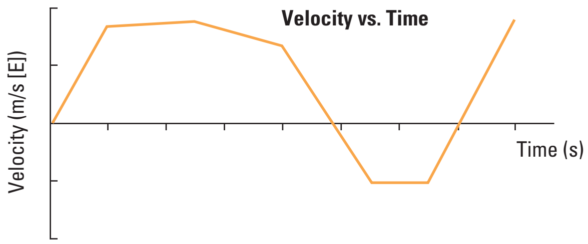
For the velocity-time graph of a ball thrown up in the air (Figure 1.44), what is the net displacement of the ball?



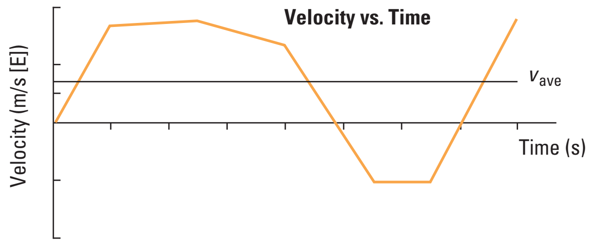
▲ **Figure 1.44**

Average Velocity from Velocity-time Graphs

Objects rarely travel at constant velocity. Think of your journey to school today. Whether you travelled by car or bus, rode a bike, or walked, stop signs, traffic lights, corners, and obstacles caused a variation in your velocity, or rate of travel. If you describe your motion to a friend, you can use a series of instantaneous velocities. The more time instances you use to record your motion, the more details about your trip you can communicate to your friend (Figure 1.45(a)). However, if you were to use the equation $\vec{v} = \dfrac{\Delta \vec{d}}{\Delta t}$ and substitute your total displacement for $\Delta \vec{d}$ and your total time of travel for Δt, you would lose most of the details of your journey. You would obtain a value for your average velocity, $\vec{v}_{\text{ave}}$ (Figure 1.45(b)).



▲ **Figure 1.45(a)** By using a series of instantaneous velocities at the given times, you can precisely describe your journey.



▲ **Figure 1.45(b)** The straight line represents the average velocity of the journey. It describes your journey but the detail of the motions is lost.

If you need to obtain an average velocity value from a velocity-time graph, recall that displacement, $\Delta \vec{d}$, is the area under the graph.

To find average velocity, determine the area under the velocity-time graph and divide it by the total time. To calculate average velocity when given different displacements over different time intervals, simply add the total displacement and divide by the total time, as shown in the next example.

Example 1.7

Find the average velocity of a student who jogs 750 m [E] in 5.0 min, does static stretches for 10.0 min, and then runs another 3.0 km [E] in 30.0 min.

Given

Choose east to be positive. Convert kilometres to metres.

$\Delta\vec{d}_1 = 750 \text{ m [E]} = +750 \text{ m}$

$\Delta t_1 = 5.0 \text{ min}$

$\Delta\vec{d}_2 = 0 \text{ m}$

$\Delta t_2 = 10.0 \text{ min}$

$\Delta\vec{d}_3 = 3.0 \text{ km [E]}$

$= +3.0 \cancel{\text{km}} \times \dfrac{1000 \text{ m}}{1 \cancel{\text{km}}} = +3000 \text{ m}$

$\Delta t_3 = 30.0 \text{ min}$

Required

average velocity ($\vec{v}_{\text{ave}}$)

Analysis and Solution

First add the displacement values.

$$\begin{aligned}\Delta\vec{d}_{\text{total}} &= \Delta\vec{d}_1 + \Delta\vec{d}_2 + \Delta\vec{d}_3 \\ &= +750 \text{ m} + 0 \text{ m} + 3000 \text{ m} \\ &= +3750 \text{ m}\end{aligned}$$

Then add the time intervals and convert to seconds. The total time elapsed is

$$\begin{aligned}\Delta t_{\text{total}} &= \Delta t_1 + \Delta t_2 + \Delta t_3 \\ &= 5.0 \text{ min} + 10.0 \text{ min} + 30.0 \text{ min} \\ &= (45.0 \cancel{\text{min}})\left(60 \frac{\text{s}}{\cancel{\text{min}}}\right) \\ &= 2700 \text{ s}\end{aligned}$$

Average velocity equals total displacement divided by total time elapsed:

$$\begin{aligned}\vec{v}_{\text{ave}} &= \frac{\Delta\vec{d}}{\Delta t} \\ &= \frac{+3750 \text{ m}}{2700 \text{ s}} \\ &= +1.4 \text{ m/s}\end{aligned}$$

Since the answer is positive, the direction is east.

Paraphrase

The student's average velocity is, therefore, 1.4 m/s [E].

Practice Problems

1. A person runs 10.0 m [E] in 2.0 s, then 5.0 m [E] in 1.5 s, and finally 30.0 m [W] in 5.0 s. Find the person's average velocity.
2. Person A runs the 100-m dash in 9.84 s and then tags person B, who runs 200 m in 19.32 s. Person B then tags an out-of-shape person C, who runs 400 m in 1.90 min. Find the average velocity for the trio. Compare it to each individual's average velocity. Assume they are all running in a straight line.

Answers

1. 1.8 m/s [W]
2. 4.89 m/s [forward]

   A: 10.2 m/s [forward]; faster than the average velocity for the trio

   B: 10.4 m/s [forward]; faster than the average velocity for the trio

   C: 3.51 m/s [forward]; slower than the average velocity for the trio

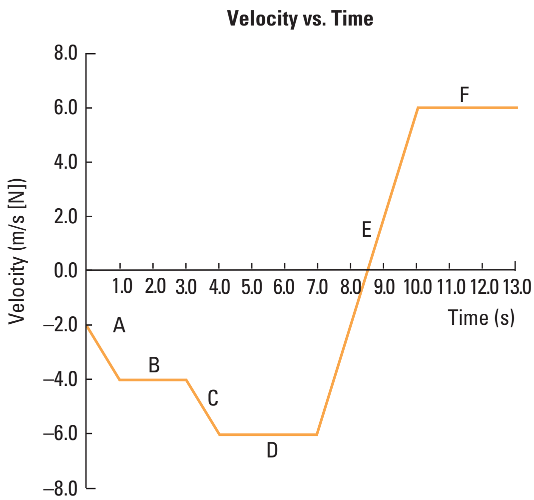
As you have seen, velocity-time graphs are very useful. They provide the following information:

- Reading the velocity-time graph gives you instantaneous velocity values.
- Finding the slope of a velocity-time graph gives you an object's acceleration.
- The area under a velocity-time graph gives you the object's displacement.
- You can also determine the average velocity of an object over a time interval from a velocity-time graph.

Example 1.8 shows you how to obtain information about an object's velocity and acceleration.

Example 1.8

A bird starts flying south. Its motion is described in the velocity-time graph in Figure 1.46.



▲ **Figure 1.46**

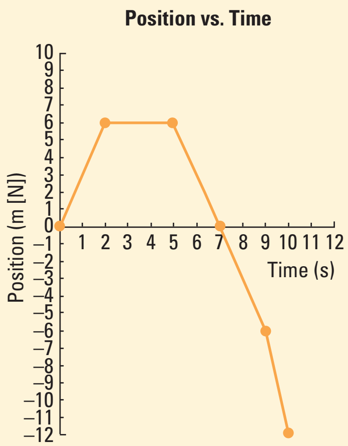
From the graph, determine
(a) whether acceleration is positive, negative, or zero for each section
(b) the value of the acceleration where it is not zero
(c) when the bird changes direction

Analysis and Solution

Consider north to be the positive direction.
(a) Acceleration is the slope of each section of the graph.
A: Final velocity is more negative than the initial velocity, as the bird is speeding up in the south direction. So the slope of this line is negative. The bird's acceleration is negative.

Practice Problems

1.



(a) Describe the motion of the object from the graph above.
(b) Draw the corresponding velocity-time graph.
(c) Determine the object's displacement.
(d) When is the object stopped?

B: Acceleration is zero because the slope is zero (the graph is a horizontal line).

C: Acceleration is negative because the slope of the line is negative (as in section A).

D: Acceleration is zero because the slope of the line is zero (as in section B).

E: Final velocity is positive because the bird is now flying north. So the slope of this line is positive. The bird's acceleration is positive.

F: Acceleration is zero because the slope of the line is zero.

(b) Acceleration is not zero for sections A, C, and E.

$$\text{A: } \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{-4.0\ \frac{\text{m}}{\text{s}} - \left(-2.0\ \frac{\text{m}}{\text{s}}\right)}{1.0\ \text{s} - 0.0\ \text{s}}$$

$$= -2.0\ \text{m/s}^2$$

$$= 2.0\ \text{m/s}^2\ [\text{S}]$$

$$\text{C: } \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{-6.0\ \frac{\text{m}}{\text{s}} - \left(-4.0\ \frac{\text{m}}{\text{s}}\right)}{4.0\ \text{s} - 3.0\ \text{s}}$$

$$= -2.0\ \text{m/s}^2$$

$$= 2.0\ \text{m/s}^2\ [\text{S}]$$

$$\text{E: } \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{+6.0\ \frac{\text{m}}{\text{s}} - \left(-6.0\ \frac{\text{m}}{\text{s}}\right)}{10.0\ \text{s} - 7.0\ \text{s}}$$

$$= \frac{+12.0\ \frac{\text{m}}{\text{s}}}{3.0\ \text{s}}$$

$$= +4.0\ \text{m/s}^2$$

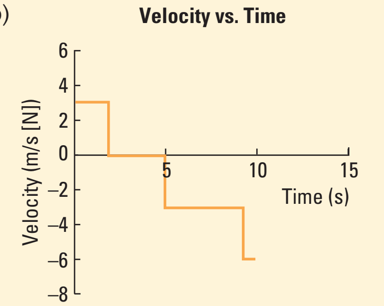
$$= 4.0\ \text{m/s}^2\ [\text{N}]$$

(c) The bird changes direction at 8.5 s — it crosses the time axis at this instant.

Answers

1. (a) 3 m/s for 2 s, rest for 3 s, −3 m/s for 4 s, −6 m/s for 1 s

(b)



(c) −12 m

(d) 2–5 s

The next example shows you how to use areas to find the displacement of the bird and its average velocity from a velocity-time graph.

Example 1.9

From the graph in Figure 1.46, determine
(a) the displacement for each section
(b) the displacement for the entire flight
(c) the average velocity of the flight

Analysis and Solution

(a) Displacement is the area between the graph and the time axis.

$$\text{A:}\quad A = l \times w + \frac{1}{2}bh$$

$$\Delta\vec{d} = (-2.0\ \frac{\text{m}}{\cancel{\text{s}}})(1.0\ \cancel{\text{s}}) + \frac{1}{2}(1.0\ \cancel{\text{s}})(-2.0\ \frac{\text{m}}{\cancel{\text{s}}})$$

$$= -3.0\ \text{m}$$

$$\text{B:}\quad A = l \times w$$

$$\Delta\vec{d} = (-4.0\ \frac{\text{m}}{\text{s}})(3.0\ \text{s} - 1.0\ \text{s})$$

$$= -8.0\ \text{m}$$

$$\text{C:}\quad A = l \times w + \frac{1}{2}bh$$

$$\Delta\vec{d} = (-4.0\ \frac{\text{m}}{\cancel{\text{s}}})(1.0\ \cancel{\text{s}}) + \frac{1}{2}(1.0\ \cancel{\text{s}})(-2.0\ \frac{\text{m}}{\cancel{\text{s}}})$$

$$= -5.0\ \text{m}$$

$$\text{D:}\quad A = l \times w$$

$$\Delta\vec{d} = (-6.0\ \frac{\text{m}}{\text{s}})(7.0\ \text{s} - 4.0\ \text{s})$$

$$= -18\ \text{m}$$

$$\text{E:}\quad A = \frac{1}{2}bh + \frac{1}{2}bh$$

$$\Delta\vec{d} = \frac{1}{2}(1.5\ \cancel{\text{s}})\left(-6.0\ \frac{\text{m}}{\cancel{\text{s}}}\right) + \frac{1}{2}(1.5\ \cancel{\text{s}})\left(+6.0\ \frac{\text{m}}{\cancel{\text{s}}}\right) = 0.0\ \text{m}$$

$$\text{F:}\quad A = l \times w$$

$$\Delta\vec{d} = (+6.0\ \frac{\text{m}}{\cancel{\text{s}}})(3.0\ \cancel{\text{s}})$$

$$= +18\ \text{m}$$

(b) Add all the displacements calculated in (a). The displacement over the entire flight is –16 m. Since north is positive, the displacement is 16 m [S].

(c)
$$\vec{v}_{\text{ave}} = \frac{\Delta\vec{d}_{\text{T}}}{\Delta t}$$

$$= \frac{-16\ \text{m}}{13.0\ \text{s}}$$

$$= -1.2\ \text{m/s}$$

North is positive, so the average velocity for the flight is 1.2 m/s [S].

Practice Problems

1. In your notebook, redraw the graph in Example 1.8 Practice Problem 1, but label the vertical axis "velocity (m/s [N])".
   (a) Find the displacements for 0–2 s, 2–5 s, 5–7 s, 7–9 s, and 9–10 s.
   (b) Find the object's total displacement.
   (c) Find the average velocity of the object.

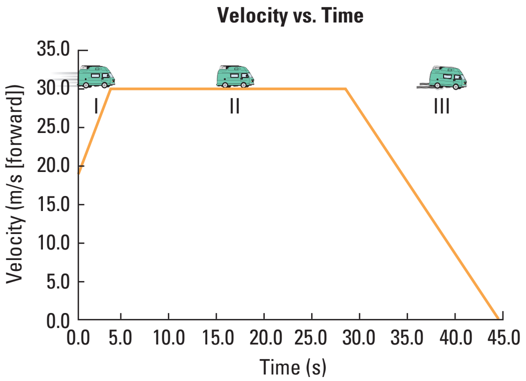
Answers

1. (a) 6 m [N], 18 m [N], 6 m [N], −6 m [N], −9 m [N]
   (b) 15 m [N]
   (c) 1.5 m/s [N]

Drawing Position-time and Acceleration-time Graphs from Velocity-time Graphs

In this section, you have learned how to use a velocity-time graph to calculate displacement. It is also useful to know how to draw position-time and acceleration-time graphs when given a velocity-time graph.

Consider the following trip. A family travelling from Calgary to go camping in Banff National Park moves at 18.0 m/s [forward] in a camper van. The van accelerates for 4.0 s until it reaches a velocity of 30.0 m/s [forward]. It continues to travel at this velocity for 25.0 s. When approaching a check stop, the driver brakes, bringing the vehicle to a complete stop in 15.0 s. The velocity-time graph for the trip is given in Figure 1.47.



◀ **Figure 1.47** The complete graph of the van's motion

The next example shows you how to create an acceleration-time graph from a velocity-time graph.

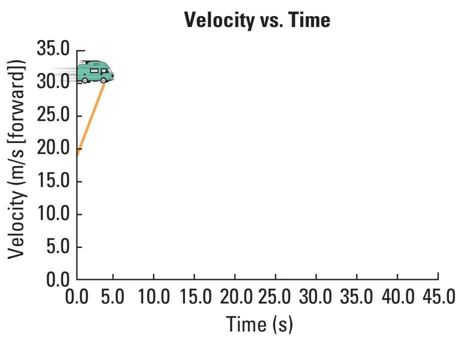
Example 1.10

Use the velocity-time graph in Figure 1.47 to draw the corresponding acceleration-time graph.

Analysis and Solution

To find acceleration, calculate the slope for each section of the graph. The velocity-time graph has three distinct sections. The slope in each section is constant. Consider forward to be positive.

Section 1
Time: 0.0 s to 4.0 s
$t_i = 0.0$ s
$v_i = +18.0$ m/s
$t_f = 4.0$ s
$v_f = +30.0$ m/s

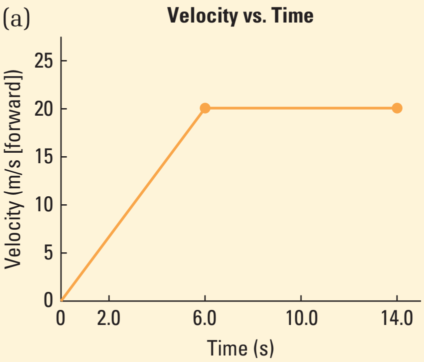


▲ **Figure 1.48(a)**

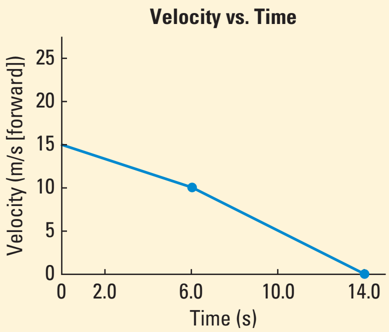
Practice Problems

1. For each velocity-time graph below, draw the corresponding acceleration-time graph.

(a)

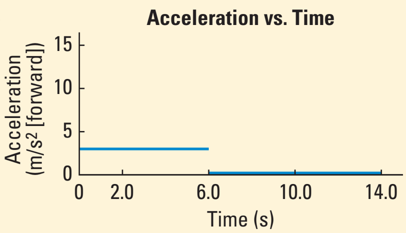


(b)

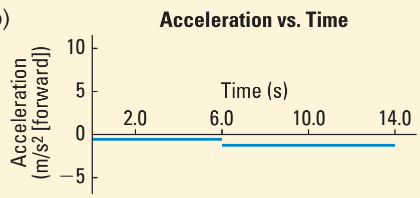


Answers

1. (a)



(b)



$$\text{slope} = \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{+30.0 \text{ m/s} - (+18.0 \text{ m/s})}{4.0 \text{ s} - 0.0 \text{ s}}$$

$$= +3.0 \text{ m/s}^2$$

Section II

Time: 4.0 s to 29.0 s

$t_i = 4.0$ s

$v_i = +30.0$ m/s

$t_f = 4.0 \text{ s} + 25.0 \text{ s}$

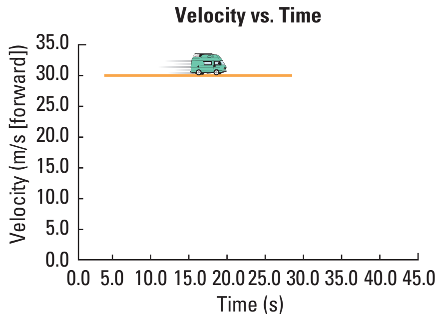
$= 29.0$ s

$v_f = +30.0$ m/s

$$\text{slope} = \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

$$= \frac{+30.0 \text{ m/s} - (+30.0 \text{ m/s})}{29.0 \text{ s} - 4.0 \text{ s}}$$

$$= 0.0 \text{ m/s}^2$$



▲ **Figure 1.48(b)**

Section III

Time: 29.0 s to 44.0 s

$t_i = 29.0$ s

$v_i = +30.0$ m/s

$t_f = 29.0 \text{ s} + 15.0 \text{ s}$

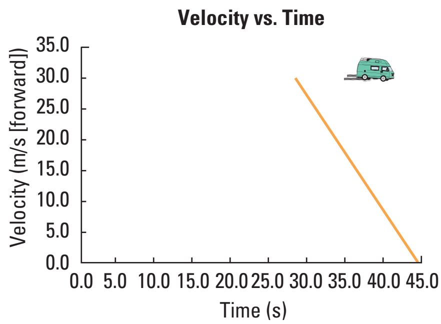
$= 44.0$ s

$v_f = 0.0$ m/s

$$\text{slope} = \vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

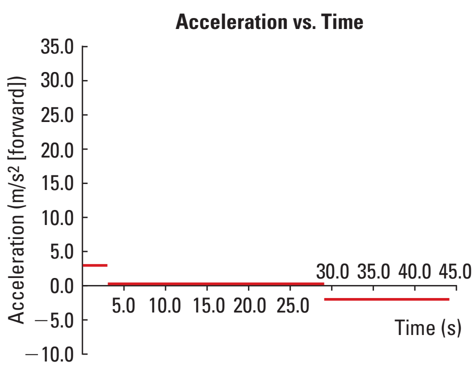
$$= \frac{0.0 \text{ m/s} - (+30.0 \text{ m/s})}{44.0 \text{ s} - 29.0 \text{ s}}$$

$$= -2.0 \text{ m/s}^2$$



▲ **Figure 1.48(c)**

Now plot the values on the acceleration-time graph (Figure 1.49). Each section of the graph is a horizontal line because acceleration is constant (uniform).



◀ **Figure 1.49**

The next example shows you how to use a velocity-time graph to generate a position-time graph.

Example 1.11

Sketch a position-time graph from the velocity-time graph in Figure 1.47.

Analysis and Solution

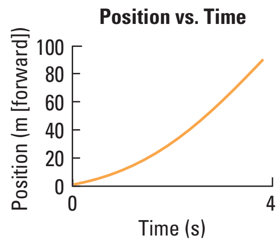
To sketch the position-time graph, find the area under the velocity-time graph. Consider forward to be positive. In the first part of the velocity-time graph (0.0–4.0 s), area (displacement) is a rectangle and a triangle. The displacement is positive because the areas are above the time-axis.

$$A = l \times w + \frac{1}{2}bh$$

$$\Delta\vec{d} = +(18.0\ \text{m/s})(4.0\ \text{s}) + \frac{1}{2}(4.0\ \text{s})(30.0\ \text{m/s} - 18.0\ \text{m/s})$$

$$= +96\ \text{m}$$

Since the velocity-time graph in this section has a positive slope, the car has positive acceleration, so the corresponding position-time graph is a parabola that curves upward. On the position-time graph, sketch a curve from the origin to the point $t = 4.0$ s and $\vec{d} = +96$ m (Figure 1.50(a)).



▲ **Figure 1.50(a)**

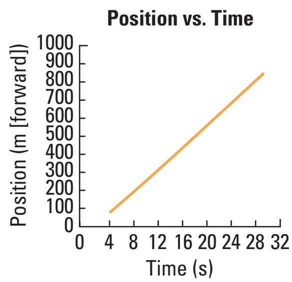
In the second part of the velocity-time graph (4.0–29.0 s), displacement is a rectangle. It is positive since the area is above the time-axis.

$$A = l \times w$$

$$\Delta\vec{d} = +(30.0\ \text{m/s})(29.0\ \text{s} - 4.0\ \text{s})$$

$$= +750\ \text{m}$$

Since the velocity-time graph has zero slope in this section, the car moves with constant velocity and the position-time graph is a straight line with a positive slope that extends from $t = 4.0$ s and $\vec{d} = +96$ m to $t = 29.0$ s and $\vec{d} = +96\ \text{m} + 750\ \text{m}$ $= +846$ m (See Figure 1.50(b).)



▲ **Figure 1.50(b)**

In the third part of the velocity-time graph (29.0–44.0 s), displacement is a triangle. It is positive since the area is above the time-axis.

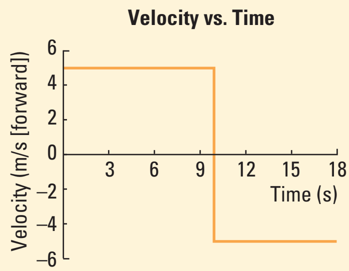
$$A = \frac{1}{2}bh$$

$$\Delta\vec{d} = +\frac{1}{2}(44.0\ \text{s} - 29.0\ \text{s})(30.0\ \text{m/s})$$

$$= +225\ \text{m}$$

Practice Problems

1.



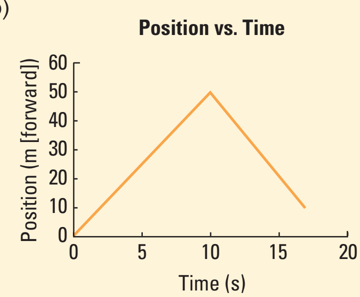
(a) Describe the motion of the object illustrated above. Calculate its total displacement.

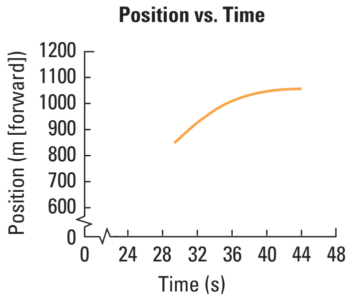
(b) Draw the corresponding position-time graph.

Answers

1. (a) travels with uniform motion, changes direction at 10 s, and travels with uniform motion; total displacement: 10 m [forward]

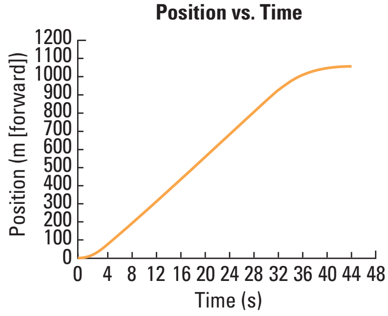
(b)





▲ **Figure 1.50(c)**

Since the velocity-time graph has a negative slope, the car undergoes negative acceleration, so the slopes of the tangents of the position-time graph decrease (approach zero). The position-time graph is a parabola that curves down,
from $t = 29.0$ s and
$\vec{d} = +846$ m to
$t = 44.0$ s and
$\vec{d} = +846 \text{ m} + 225 \text{ m}$
$= +1071$ m
(See Figure 1.50(c).)
The resulting position-time graph is shown in Figure 1.51.



▲ **Figure 1.51**

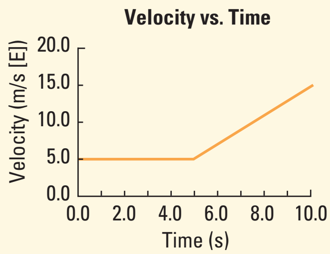
Concept Check

If north is positive, sketch position-time, velocity-time, and acceleration-time graphs for an object
(a) speeding up and going north
(b) slowing down and going north
(c) speeding up and going south
(d) slowing down and going south

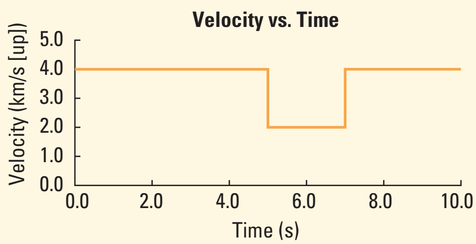
1.4 Check and Reflect

Knowledge

1. On a ticker tape, how can you distinguish between uniform and uniformly accelerated motion?
2. Use the terms "displacement" and "velocity" to describe how uniformly accelerated motion differs from uniform motion.
3. What is the relationship between the slope of a position-time graph and velocity?
4. Compare the shape of a position-time graph for uniform motion with a position-time graph representing uniformly accelerated motion.
5. What is the relationship between the slope of a velocity-time graph and acceleration?
6. If a velocity-time graph is a straight line with a non-zero slope, what kind of motion is the object undergoing?
7. Determine the displacement of the object whose motion is described by the following graph.



8. Calculate displacement from the velocity-time graph below.



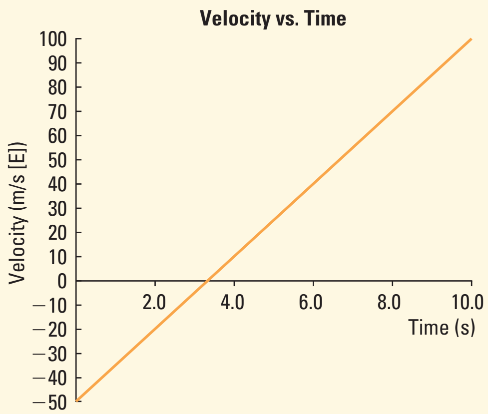
9. Describe the velocity-time graph for an object undergoing negative acceleration.

10. What quantity of motion can be determined from the area under a velocity-time graph?

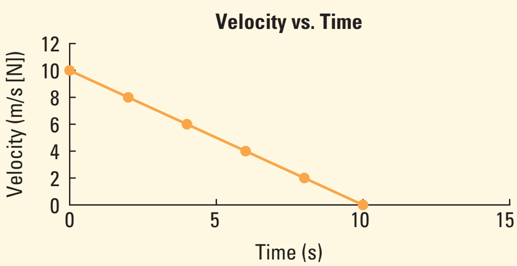
11. Compare and contrast the shape of a velocity-time graph for an object experiencing uniform motion with one experiencing uniformly accelerated motion.

12. Describe the acceleration-time graph of a car travelling forward and applying its brakes.

13. Calculate the acceleration of an object using the velocity-time graph below.



14. Construct an acceleration-time graph using the graph given below.



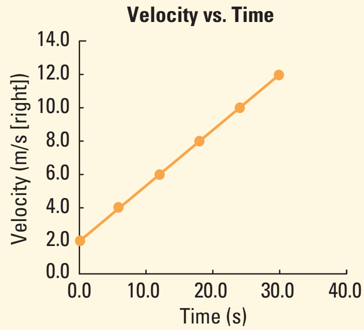
Applications

15. A motorbike increases its velocity from 20.0 m/s [W] to 30.0 m/s [W] over a distance of 200 m. Find the acceleration and the time it takes to travel this distance.

16. (a) While driving north from Lake Louise to Jasper, you travel 75 min at a velocity of 70 km/h [N] and another 96 min at 90 km/h [N]. Calculate your average velocity.

    (b) Create a graph for the question and check your answer using graphing techniques.

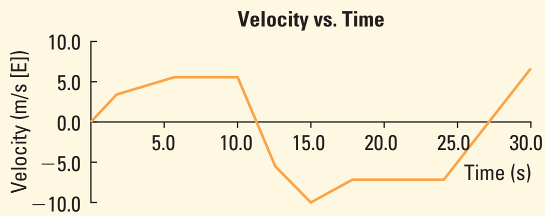
17. Determine acceleration from the velocity-time graph given below.



18. A truck travelling forward at 14.0 m/s accelerates at 1.85 m/s^2 for 6.00 s. It then travels at the new speed for 35.0 s, when a construction zone forces the driver to push on the brakes, providing an acceleration of −2.65 m/s^2 for 7.0 s. Draw the resulting velocity-time and position-time graphs for this motion.

Extension

19. Describe the motion of the object illustrated in the graph below.



e TEST

To check your understanding of velocity-time graphs follow the eTest links at www.pearsoned.ca/school/physicssource.

1.5 The Kinematics Equations

*e*TECH

Study the physics of jet takeoffs by visiting www.pearsoned.ca/school/physicssource and viewing the simulation.

A cylindrical piston the length of a football field controls the launch of a fighter plane from the deck of a carrier ship (Figure 1.52). Too much pressure and the nose gear is ripped off; too little pressure and the plane crashes into the ocean. This propulsion system accelerates a 20 000-kg plane from rest to 74 m/s (266 km/h) in just 2.0 s!

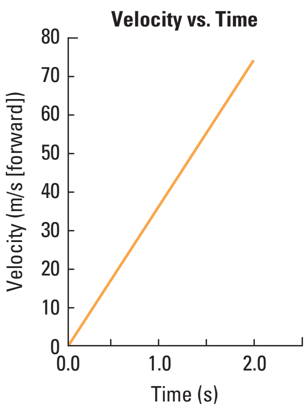
▶ **Figure 1.52** Analyzing complex motions requires many calculations, some of which involve using the kinematics equations you will study in this section.

To determine the crucial values required for launching a plane, such as flight deck length, final velocity, and acceleration, physicists and engineers use kinematics equations similar to the ones you will know by the end of this section. In this section, you will practise your analytical skills by learning how to derive the kinematics equations from your current knowledge of graphs and then apply these equations to analyze complex motions such as airplane launches.

Concept Check

Create a summary chart for the information you can gather by analyzing position-time, velocity-time, and acceleration-time graphs. Use the headings "Graph Type", "Reading the Graph", "Slope", and "Area".

Consider the airplane taking off from a moving aircraft carrier (Figure 1.52). The plane must reach its takeoff speed before it comes to the end of the carrier's runway. If the plane starts from rest, the velocity-time graph representing the plane's motion is shown in Figure 1.53. Notice that the slope of the graph is constant. By checking the units on the graph, you know that the slope represents acceleration: $\frac{\text{rise}}{\text{run}} = \frac{\text{m/s}}{\text{s}} = \text{m/s}^2$. In this case, the acceleration is constant (uniform). Therefore, the velocity-time graph is a straight line.



▲ **Figure 1.53** The slope of this velocity-time graph represents the plane's acceleration.

From Figure 1.53, you can derive the first kinematics equation:

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{t_f - t_i}$$

This equation can also be written as

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$$

The next example shows you how to apply this equation to solve a problem.

Example 1.12

A hybrid car with an initial velocity of 10.0 m/s [E] accelerates at 3.0 m/s^2 [E]. How long will it take the car to acquire a final velocity of 25.0 m/s [E]?

Given

Designate east as the positive direction.

$\vec{v}_i$ = 10.0 m/s [E] = +10.0 m/s

$\vec{v}_f$ = 25.0 m/s [E] = +25.0 m/s

$\vec{a}$ = 3.0 m/s^2 [E] = +3.0 m/s^2

Required

time (Δt)

Analysis and Solution

Use the equation

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

Since you are dividing by a vector, and initial and final velocities and acceleration are in the same direction, use the scalar form of the equation. Isolate Δt and solve.

$$\begin{aligned}\Delta t &= \frac{v_f - v_i}{a} \\ &= \frac{25 \text{ m/s} - 10 \text{ m/s}}{3.0 \text{ m/s}^2} \\ &= \frac{15 \frac{\cancel{\text{m}}}{\cancel{\text{s}}}}{3.0 \frac{\cancel{\text{m}}}{\text{s}^{\cancel{2}}}} \\ &= 5.0 \text{ s}\end{aligned}$$

Paraphrase

It will take the car 5.0 s to reach a velocity of 25.0 m/s [E].

Practice Problems

1. A motorcycle with an initial velocity of 6.0 m/s [E] accelerates at 4.0 m/s^2 [E]. How long will it take the motorcycle to reach a final velocity of 36.0 m/s [E]?
2. An elk moving at a velocity of 20 km/h [N] accelerates at 1.5 m/s^2 [N] for 9.3 s until it reaches its maximum velocity. Calculate its maximum velocity, in km/h.

Answers

1. 7.5 s
2. 70 km/h [N]

PHYSICS INSIGHT

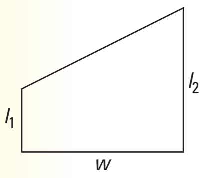
The mathematics of multiplying vectors is beyond this text and division of vectors is not defined. So, when multiplying and dividing vectors, use the scalar versions of the kinematics equations.

As you know from section 1.4, you can calculate the area under a velocity-time graph. By checking the units, you can verify that the area represents displacement:

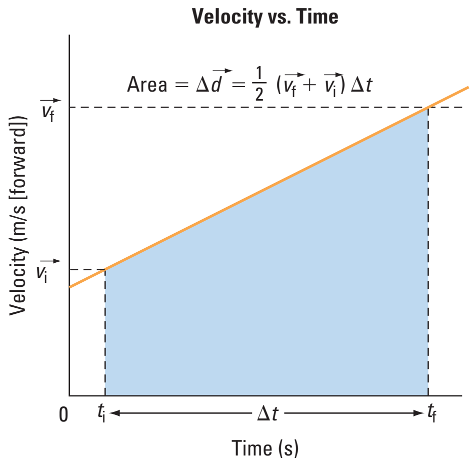
$$l \times w = \frac{\text{m}}{\cancel{\text{s}}} \times \cancel{\text{s}} = \text{m}$$

PHYSICS INSIGHT

The area of a trapezoid is given by $\frac{1}{2}(l_1 + l_2)w$.



To calculate the displacement (area) from the velocity-time graph in Figure 1.54, you can use the formula for the area of a trapezoid, $A = \frac{1}{2}(l_1 + l_2)w$, which is simply the average of the parallel sides multiplied by the base.



▲ **Figure 1.54** Use the area under the velocity-time graph to derive the equation $\vec{d} = \vec{v}_{ave}\Delta t$.

The second kinematics equation is

$$\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$$

where $l_1 = \vec{v}_i$, $l_2 = \vec{v}_f$, and $w = \Delta t$. The next example shows you how to apply this equation.

Example 1.13

A cattle train travelling west at 16.0 m/s is brought to rest in 8.0 s. Find the displacement of the cattle train while it is coming to a stop. Assume uniform acceleration.

Given

Designate west as the positive direction.

$\vec{v}_i = 16.0$ m/s [W] $= +16.0$ m/s

$\vec{v}_f = 0$ m/s [W] $= 0$ m/s

$\Delta t = 8.0$ s

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Use the equation $\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$ and solve for $\Delta\vec{d}$.

Practice Problems

1. A hound running at a velocity of 16 m/s [S] slows down uniformly to a velocity of 4.0 m/s [S] in 4.0 s. What is the displacement of the hound during this time?
2. A ball moves up a hill with an initial velocity of 3.0 m/s. Four seconds later, it is moving down the hill at 9.0 m/s. Find the displacement of the ball from its initial point of release.

Answers

1. 40 m [S]
2. −12 m

$$\Delta\vec{d} = \frac{1}{2}(+16.0 \text{ m/s} + 0 \text{ m/s})(8.0 \text{ s})$$

$$= \left(+8.0\ \frac{\text{m}}{\cancel{\text{s}}}\right)(8.0\ \cancel{\text{s}})$$

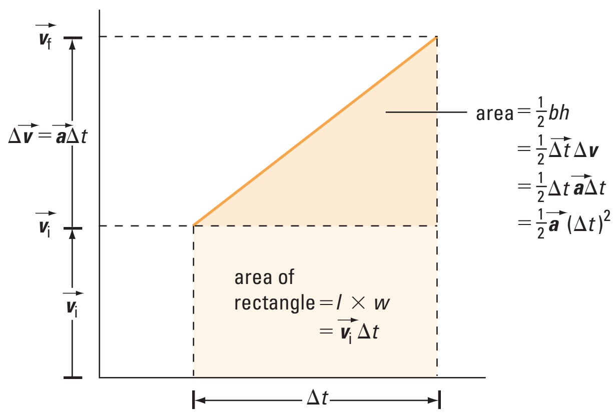
$$= +64 \text{ m}$$

The sign is positive, so the train's direction is west.

Paraphrase

The cattle train travels 64 m [W] before it stops.

You can also calculate the area under a velocity-time graph by considering that the total area under the graph is made up of a triangle and a rectangle (Figure 1.55).



▲ **Figure 1.55** You can divide the area under the velocity-time graph into a triangle and a rectangle.

In Figure 1.55, the area of a rectangle represents the displacement of an object travelling with a constant velocity, $\vec{v}_i$. The height of the rectangle is $\vec{v}_i$ and the base is Δt. Therefore, the area of the rectangle is equal to $\vec{v}_i\Delta t$. The area of the triangle represents the additional displacement resulting from the change in velocity. The height of the triangle is $\vec{v}_f - \vec{v}_i = \Delta\vec{v}$ and the base is Δt. The area of the triangle is equal to $\frac{1}{2}\Delta t(\Delta\vec{v})$. But $\Delta\vec{v} = \vec{a}\Delta t$. Therefore, the area of the triangle is equal to $\frac{1}{2}(\Delta t)(\vec{a}\Delta t) = \frac{1}{2}\vec{a}(\Delta t)^2$. Add both displacements to obtain

$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

The next example shows you how to apply the third kinematics equation.

info **BIT**

The fastest time of covering 1.6 km while flipping tiddly winks — 52 min 10 s — was achieved by E. Wynn and J. Culliongham (UK) on August 31, 2002. Their speed was 1.8 km/h. Using this data and the displacement equation below, verify that they did indeed travel a distance of 1.6 km.

PHYSICS INSIGHT

$\Delta(t^2) = t_f^2 - t_i^2$, whereas $(\Delta t)^2 = (t_f - t_i)^2$.

Example 1.14

A golf ball that is initially travelling at 25 m/s hits a sand trap and slows down with an acceleration of $-20\ \text{m/s}^2$. Find its displacement after 2.0 s.

Given

Assign a positive direction for forward and a negative direction for backward.

$v_i = +25\ \text{m/s}$

$\vec{a} = -20\ \text{m/s}^2$

$\Delta t = 2.0\ \text{s}$

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Use the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$ to solve for $\Delta\vec{d}$.

$$\Delta\vec{d} = \left(+25\ \frac{\text{m}}{\cancel{\text{s}}}\right)(2.0\ \cancel{\text{s}}) + \frac{1}{2}\left(-20\ \frac{\text{m}}{\cancel{\text{s}^2}}\right)(2.0\ \cancel{\text{s}})^{\cancel{2}}$$

$$= +50\ \text{m} + (-40\ \text{m})$$

$$= +10\ \text{m}$$

The sign is positive, so the direction is forward.

Paraphrase

The displacement of the golf ball is 10 m [forward].

Practice Problems

1. A skier is moving down a uniform slope at 3.0 m/s. If the acceleration down the hill is $4.0\ \text{m/s}^2$, find the skier's displacement after 5.0 s.
2. A motorcycle travelling at 100 km/h on a flat road applies the brakes at $0.80\ \text{m/s}^2$ for 1.0 min. How far did the motorcycle travel during this time?

Answers

1. 65 m [down]
2. 2.3×10^2 m

To obtain the fourth kinematics equation, derive the value of a required variable in one equation, substitute the derived value into the second equation, and simplify.

Start with $\vec{a} = \dfrac{\vec{v}_f - \vec{v}_i}{\Delta t}$

Isolate $\vec{v}_i$.

$$\vec{v}_i = \vec{v}_f - \vec{a}\Delta t$$

Then substitute $\vec{v}_f - \vec{a}\Delta t$ for $\vec{v}_i$ into the equation $\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$. The equation becomes $\Delta\vec{d} = \frac{1}{2}(\vec{v}_f - \vec{a}\Delta t + \vec{v}_f)\Delta t$.

This equation simplifies to

$$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$$

Apply this equation in the next example.

info **BIT**

The fastest lava flow ever recorded was 60 km/h in Nyiragongo (Democratic Republic of Congo) on January 10, 1977. At this speed, how far would the lava travel in 2 h 30 min?

Example 1.15

▲ **Figure 1.56**

A speedboat slows down at a rate of 5.0 m/s² and comes to a stop (Figure 1.56). If the process took 15 s, find the displacement of the boat.

Given

Let forward be the positive direction.

$\vec{v}_f = 0.0$ m/s (because the boat comes to rest)

$\Delta t = 15$ s

$\vec{a} = -5.0$ m/s² (Acceleration is negative because the boat is slowing down, so its sign must be opposite to that of velocity (positive).)

Required

displacement ($\Delta\vec{d}$)

Analysis and Solution

Use the equation $\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$ to solve for $\Delta\vec{d}$.

$$\Delta\vec{d} = (0.0\ \text{m/s})(15\ \text{s}) - \frac{1}{2}(-5.0\ \text{m/s}^2)(15\ \text{s})^2$$

$$= +562.5\ \text{m}$$

$$= +5.6 \times 10^2\ \text{m}$$

The sign is positive, so the direction of displacement is forward.

Paraphrase

The displacement of the speedboat is 5.6×10^2 m [forward].

Practice Problems

1. If the arresting device on an aircraft carrier stops a plane in 150 m with an acceleration of −15 m/s², find the time the plane takes to stop.
2. The 1968 Corvette took 6.2 s to accelerate to 160 km/h [N]. If it travelled 220 m [N], find its acceleration.

Answers

1. 4.5 s
2. 2.9 m/s² [N]

Deriving the fifth and last kinematics equation involves using the difference of squares, another math technique.

Isolate Δt in the equation $\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$. Remember that, when multiplying or dividing vectors, use the scalar form of the equation:

$$\Delta t = \frac{v_f - v_i}{a}$$

Then substitute the expression for Δt into $\Delta d = \frac{1}{2}(v_i + v_f)\Delta t$:

$$\Delta d = \frac{1}{2}(v_i + v_f)\left(\frac{v_f - v_i}{a}\right)$$

$$\Delta d = \frac{1}{2}\left(\frac{v_f^2 - v_i^2}{a}\right)$$

PHYSICS INSIGHT

Recall that the difference of squares is $(a + b)(a - b) = a^2 - b^2$

info **BIT**

A tortoise covered 5.48 m in 43.7 s at the National Tortoise Championships in Tickhill, UK, to set a world record on July 2, 1977. It was moving at 0.45 km/h.

The more standard form of the fifth kinematics equation is

$$v_f^2 = v_i^2 + 2a\Delta d$$

This equation is applied in the next example.

Example 1.16

A bullet accelerates the length of the barrel of a gun (0.750 m) with a magnitude of 5.35×10^5 m/s². With what speed does the bullet exit the barrel?

Given

$a = 5.35 \times 10^5 \text{ m/s}^2$
$d = 0.750 \text{ m}$

Required

final speed (v_f)

Analysis and Solution

Use the equation $v_f^2 = v_i^2 + 2a\Delta d$. Since the bullet starts from rest, $v_i = 0$ m/s.

$$\begin{aligned} v_f^2 &= (0 \text{ m/s})^2 + 2(5.35 \times 10^5 \text{ m/s}^2)(0.750 \text{ m}) \\ &= 802\ 500 \text{ m}^2/\text{s}^2 \\ v_f &= \sqrt{802\ 500 \text{ m}^2/\text{s}^2} \\ &= 896 \text{ m/s} \end{aligned}$$

Paraphrase

The bullet leaves the barrel of the gun with a speed of 896 m/s.

Practice Problems

1. A jetliner lands on a runway at 70 m/s, reverses its engines to provide braking, and comes to a halt 29 s later.
   (a) What is the jet's acceleration?
   (b) What length of runway did the jet require to come safely to a complete stop?
2. On-ramps are designed so that motorists can move seamlessly into highway traffic. If a car needs to increase its speed from 50 km/h to 100 km/h and the engine can provide a maximum acceleration of magnitude 3.8 m/s², find the minimum length of the on-ramp.

Answers

1. (a) −2.4 m/s² [forward]
   (b) 1.0 km
2. 76 m

It is important to note that the velocity-time graph used to derive the kinematics equations has a constant slope (see Figure 1.54), so the equations derived from it are for objects undergoing *uniformly accelerated motion* (constant acceleration).

General Method of Solving Kinematics Problems

Now that you know five kinematics equations, how do you know which one to use to solve a problem? To answer this question, notice that each of the five kinematics equations has four variables. Each kinematics problem will provide you with three of these variables, as given values. The fourth variable represents the unknown value. When choosing your equation, make sure that all three known variables and the one unknown variable are represented in the equation (see Table 1.8). You may need to rearrange the equation to solve for the unknown variable.

▼ Table 1.8 The Variables in the Five Kinematics Equations

Equation	$\Delta\vec{d}$	$\vec{a}$	$\vec{v}_f$	$\vec{v}_i$	Δt
$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$		X	X	X	X
$\Delta\vec{d} = \frac{1}{2}(\vec{v}_i + \vec{v}_f)\Delta t$	X		X	X	X
$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$	X	X		X	X
$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$	X	X	X		X
$v_f^2 = v_i^2 + 2a\Delta d$	X	X	X	X	

PHYSICS INSIGHT

Remember these implied given values.
- If the object starts from rest, $\vec{v}_i = 0$.
- If the object comes to a stop, $\vec{v}_f = 0$.
- If the object experiences uniform motion, $\vec{a} = 0$.

1.5 Check and Reflect

Applications

1. How far will a humanoid robot travel in 3.0 s, accelerating at 1.0 cm/s² [forward], if its initial velocity is 5.0 cm/s [forward]?
2. What is the displacement of a logging truck accelerating from 10 m/s [right] to 20 m/s [right] in 5.0 s?
3. How far will a car travel if it starts from rest and experiences an acceleration of magnitude 3.75 m/s² [forward] for 5.65 s?
4. Determine the acceleration of a bullet starting from rest and leaving the muzzle 2.75×10^{-3} s later with a velocity of 460 m/s [forward].
5. Some aircraft are capable of accelerations of magnitude 42.5 m/s². If an aircraft starts from rest, how long will it take the aircraft to travel down the 2.6-km runway?
6. If a cyclist travelling at 14.0 m/s skids to a stop in 5.60 s, determine the skidding distance. Assume uniform acceleration.
7. Approaching a flashing pedestrian-activated traffic light, a driver must slow down to a speed of 30 km/h. If the crosswalk is 150 m away and the vehicle's initial speed is 50 km/h, what must be the magnitude of the car's acceleration to reach this speed limit?
8. A train's stopping distance, even when full emergency brakes are engaged, is 1.3 km. If the train was travelling at an initial velocity of 90 km/h [forward], determine its acceleration under full emergency braking.
9. A rocket starts from rest and accelerates uniformly for 2.00 s over a displacement of 150 m [W]. Determine the rocket's acceleration.
10. A jet starting from rest reaches a speed of 241 km/h on 96.0 m of runway. Determine the magnitude of the jet's acceleration.
11. What is a motorcycle's acceleration if it starts from rest and travels 350.0 m [S] in 14.1 s?
12. Determine the magnitude of a car's acceleration if its stopping distance is 39.0 m for an initial speed of 97.0 km/h.
13. A typical person can tolerate an acceleration of about −49 m/s² [forward]. If you are in a car travelling at 110 km/h and have a collision with a solid immovable object, over what minimum distance must you stop so as to not exceed this acceleration?
14. Determine a submarine's acceleration if its initial velocity is 9.0 m/s [N] and it travels 1.54 km [N] in 2.0 min.

e TEST

To check your understanding of the kinematics equations, follow the eTest links at www.pearsoned.ca/school/physicssource.

▲ **Figure 1.57** Amusement park rides are an application of physics.

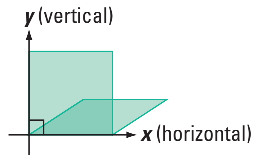
projectile motion: motion in a vertical plane

projectile: an object released or thrown into the air

1.6 Acceleration due to Gravity

Many amusement parks and midways showcase a ride based solely on acceleration due to gravity. The ride transports thrill seekers up to a dizzying height, allows them to come to rest, and then, without warning, releases them downward before coming to a controlled stop (Figure 1.57).

In the previous sections, you learned about objects that move in a horizontal plane. Many objects move in a vertical plane. Flipping a coin, punting a football, and a free throw in basketball are all examples of objects experiencing motion in a vertical plane (Figure 1.58). This type of motion is called **projectile motion**. A **projectile** is any object thrown into the air. Projectiles include objects dropped from rest; objects thrown downward or vertically upward, such as a tennis ball for service; and objects moving upward at an angle, such as a punted football. First let's consider projectile motion of an object moving straight up or down. What is the relationship between an object's mass and the speed of its fall? Do the next QuickLab to find out.



▲ **Figure 1.58** A plane has two dimensions, x and y.

1-6 QuickLab

The Bigger They Are . . .

Problem

Does mass affect how quickly an object falls?

Materials

two objects of similar size and shape but different mass, such as a marble and a ball bearing, a die and a sugar cube, a golf ball and a table tennis ball
two pans
chair

Procedure

1. Place a pan on either side of the chair.
2. Standing on the chair, release each pair of objects from the same height at the same time.
3. Listen for the objects hitting the pans.
4. Repeat steps 2 and 3 for other pairs of objects.
5. Repeat steps 2 and 3 from a higher and a lower height.

Questions

1. Did the pair of objects land at the same time?
2. How did a change in height affect how long it took the objects to drop?
3. How did a change in the objects' shape affect how long it took each pair to drop?

info BIT

In 1971, astronaut David Scott tested Galileo's theory with a feather and a hammer. With no air on the Moon, both objects hit the ground at the same time.

In the 16th century, Galileo conducted experiments that clearly demonstrated that objects falling near Earth's surface have a constant acceleration, neglecting air resistance, called the **acceleration due to gravity**. You can determine the value of the acceleration due to gravity by performing the following experiment.

Determining the Magnitude of the Acceleration due to Gravity

Required Skills
- Initiating and Planning
- Performing and Recording
- Analyzing and Interpreting
- Communication and Teamwork

Question

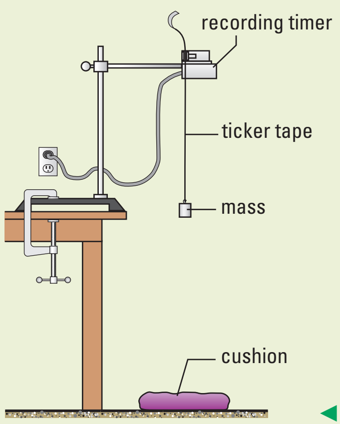
How can position-time and velocity-time graphs be used to determine the acceleration due to gravity?

Materials and Equipment

60-Hz spark timer
ticker tape
carbon disk
power supply
small mass
metre-stick or ruler
masking tape
C-clamp
retort stand
graph paper
cushion

Procedure

1. Construct a data table in your notebook for recording time and position.
2. Set up materials as shown in Figure 1.59(a), ensuring that the timer is 1.5 m above the floor.

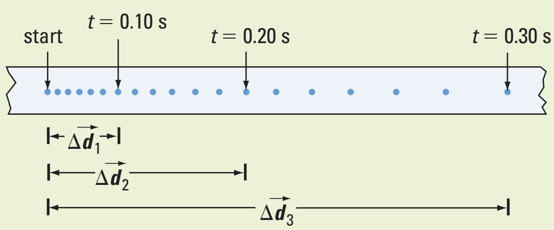


◀ **Figure 1.59(a)**

3. Attach a 1.5-m strip of ticker tape to the mass and thread the ticker tape through the spark timer.
4. Turn on the spark timer just before your partner releases the mass.
5. Repeat steps 3 and 4 for each person in your group.
6. Analyze the ticker tape by drawing a line through the first distinct dot on the tape. Label it "start". (On a 60-Hz timer, every sixth dot represents 0.10 s.) Continue labelling your ticker tape as shown in Figure 1.59(b).
7. Using a ruler, measure the position of the object at each time interval and record it in your data table.
8. Plot your collected data on a position-time graph.
9. With a sweeping motion, practise connecting the dots in a smooth curve that best fits the data.
10. Construct a data table in your notebook for recording instantaneous velocity and time.
11. Draw three tangents on the position-time graph.
12. Calculate the instantaneous velocities at these points by determining the slopes of the tangents. Record the data in your table.
13. Plot a velocity-time graph of your collected data.
14. Draw a line of best fit.
15. Calculate the acceleration experienced by the object, in m/s^2, by finding the slope of the velocity-time graph.

Analysis

1. Determine the experimental value of the magnitude of acceleration due to gravity by averaging your group's results.
2. Determine the percent error for your experimental value. Assume the theoretical magnitude of a is 9.81 m/s^2.
3. Describe the shape of the position-time graph you drew in step 9.
4. From your graph, describe the relationship between time and displacement for an accelerating object.



▲ **Figure 1.59(b)**

eLAB

For a probeware activity, go to www.pearsoned.ca/school/physicssource.

Gravity Causes Objects to Accelerate Downward

Recall the kinematics equations for accelerated motion from section 1.5:

$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

$$\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\Delta \vec{d} = \vec{v}_f \Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$$

$$v_f^2 = v_i^2 + 2a\Delta d$$

You can also apply these equations to motion in a vertical plane. Because vertical acceleration is due to gravity, $\vec{a}$ is the acceleration due to gravity, or 9.81 m/s² [down].

If you drop a golf ball from a height of 1.25 m, how long will it take for the ball to reach the ground (Figure 1.60)?

Because the ball is moving in only one direction, down, choose down to be positive for simplicity. Since the golf ball is accelerating due to gravity starting from rest,

$$\vec{v}_i = 0 \text{ and } \vec{a} = 9.81 \text{ m/s}^2 \text{ [down]} = +9.81 \text{ m/s}^2$$

The ball's displacement can be expressed as 1.25 m [down], or +1.25 m. The equation that includes all the given variables and the unknown variable is $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$. The displacement and acceleration vectors are both in the same direction, so use the scalar form of the equation to solve for time. Since $v_i = 0$,

$$\Delta d = \frac{1}{2}a\Delta t^2$$

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

$$= \sqrt{\frac{2(1.25 \text{ m})}{9.81 \frac{\text{m}}{\text{s}^2}}}$$

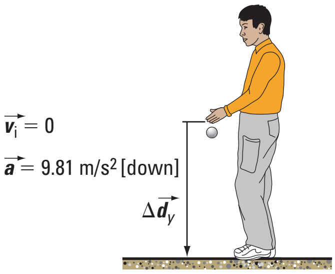
$$= 0.505 \text{ s}$$

The golf ball takes 0.505 s to reach the ground when released from a rest height of 1.25 m.

Note that the time it takes for an object to fall is directly proportional to the square root of the height it is dropped from: $\Delta t = \sqrt{\frac{2\Delta \vec{d}}{\vec{a}}}$. If there is no air resistance, the time it takes for a falling object to reach the ground depends only on the height from which it was dropped. The time does not depend on any other property of the object.

e TECH

Use graphical analysis to determine acceleration due to Earth's gravity. Go to www.pearsoned.ca/school/physicssource.



▲ **Figure 1.60** The time it takes the golf ball to hit the ground depends on the height from which it drops and on the acceleration due to gravity.

PHYSICS INSIGHT

The equations of parabolas are quadratic equations because they include a power of two, for example, $y = x^2$. The equation for the displacement of a vertical projectile is $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$.

info BIT

Without a parachute, Vesna Vulovic, a flight attendant, survived a fall of 10 160 m when the DC-9 airplane she was travelling in exploded.

1-8 QuickLab

Could You Be a Goalie for the NHL?

Problem

What is your reaction time?

Materials and Equipment

long ruler (30 cm or more)
flat surface

Procedure

1. Rest your arm on a flat surface with your wrist at the edge.
2. Ask your partner to hold the ruler vertically so that the ruler's end is just above your hand.
3. Curl your fingers so that the space between your thumb and index finger is large enough for the ruler to pass through easily.
4. Without watching your partner, ask your partner to let go of the ruler without warning.
5. Try to close your thumb and index finger on the ruler as quickly as possible.
6. Record where your hand is on the ruler.
7. Repeat steps 1–6 several times.

Questions

1. Determine the average distance the ruler falls in each of your trials.
2. Using the average distance, calculate the time.
3. An average NHL goalie has a reaction time of 0.15 s. How does your reaction time compare with your partner's?
4. Certain drugs impair reaction time. What would you expect your results in this lab to be if your reaction time were increased?

Instead of dropping an object such as a golf ball, what if you threw an object down? By throwing an object straight down, you give the object an initial vertical velocity downward. What effect does an initial velocity have on the motion of the object? The next example will show you.

Example 1.17

While cliff diving in Mexico, a diver throws a smooth, round rock straight down with an initial speed of 4.00 m/s. If the rock takes 2.50 s to land in the water, how high is the cliff?

Given
For convenience, choose down to be positive because down is the only direction of the ball's motion.

$\vec{v}_i = 4.00 \text{ m/s [down]} = +4.00 \text{ m/s}$

$\Delta t = 2.50 \text{ s}$

$\vec{a} = 9.81 \text{ m/s}^2 \text{ [down]} = +9.81 \text{ m/s}^2$

Required
height of cliff (Δd)

Analysis and Solution
The initial velocity and acceleration vectors are both in the same direction, so use the scalar form of the equation $\Delta \vec{d} = \vec{v}_i \Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$.

Practice Problems

1. If a rock takes 0.750 s to hit the ground after being thrown down from a height of 4.80 m, determine the rock's initial velocity.
2. Having scored a touchdown, a football player spikes the ball in the end zone. If the ball was thrown down with an initial velocity of 2.0 m/s from a height of 1.75 m, determine how long it is in the air.
3. An elevator moving downward at 4.00 m/s experiences an upward acceleration of 2.00 m/s² for 1.80 s. What is its velocity at the end of the acceleration and how far has it travelled?

Answers

1. 2.72 m/s [down]
2. 0.43 s
3. 0.400 m/s [down], 3.96 m

$$\begin{aligned}\Delta d &= (4.00\ \text{m/s})(2.50\ \text{s}) + \frac{1}{2}(9.81\ \text{m/s}^2)(2.50\ \text{s})^2 \\ &= 10.0\ \text{m} + 30.7\ \text{m} \\ &= 40.7\ \text{m}\end{aligned}$$

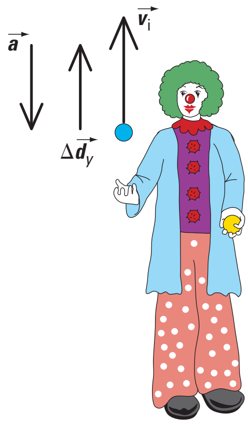
Paraphrase

The cliff is 40.7 m high.

What Goes Up Must Come Down

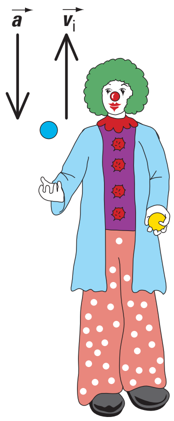
Circus clowns are often accomplished jugglers (Figure 1.61). If a juggler throws a ball upward, giving it an initial velocity, what happens to the ball (Figure 1.62)?

▲ **Figure 1.61** Juggling is an example of projectile motion.

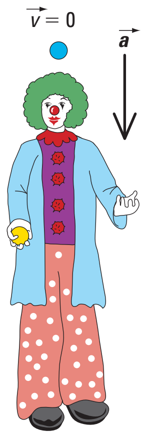


▲ **Figure 1.62** The ball's motion is called vertical projectile motion.

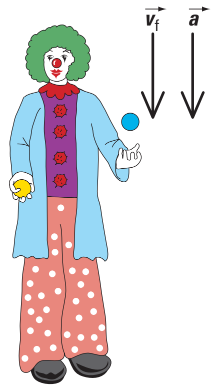
When you throw an object up, its height (displacement) increases while its velocity decreases. The decrease in velocity occurs because the object experiences acceleration downward due to gravity (Figure 1.63(a)). The ball reaches its maximum height when its vertical velocity equals zero. In other words, it stops for an instant at the top of its path (Figure 1.63(b)). When the object falls back toward the ground, it speeds up because of the acceleration due to gravity (Figure 1.63(c)).



▲ **Figure 1.63(a)**
Stage 1: Velocity and acceleration are in opposite directions, so the ball slows down.



▲ **Figure 1.63(b)**
Stage 2: The ball has momentarily stopped, but its acceleration is still 9.81 m/s² [down], which causes the ball to change direction.



▲ **Figure 1.63(c)**
Stage 3: Velocity and acceleration are in the same direction, so the ball speeds up.

The next two examples analyze different stages of the same object's motion. Example 1.18 analyzes the upward part of the motion of an object thrown upward, whereas Example 1.19 analyzes the same object's downward motion.

Example 1.18

A clown throws a ball upward at 10.00 m/s. Find
(a) the maximum height the ball reaches above its launch height
(b) the time it takes to do so

Given
Consider up to be positive.

$v_i = 10.00$ m/s [up] $= +10.00$ m/s

$\vec{a} = 9.81$ m/s² [down] $= -9.81$ m/s²

Required
(a) maximum height above launch height (Δd)
(b) time taken to reach maximum height (Δt)

Analysis and Solution
(a) When you throw an object up, as its height increases, its speed *decreases* because the object is accelerating downward due to gravity. The ball, travelling upward away from its initial launch height, reaches its maximum height when its vertical velocity is zero. In other words, the object stops for an instant at the top of its path up, so $v_f = 0.00$ m/s. To find the object's maximum height, neglecting air friction, use the equation $v_f^2 = v_i^2 + 2a\Delta d$ and substitute scalar quantities.

Practice Problem

1. The Slingshot drops riders 27 m from rest before slowing them down to a stop. How fast are they moving before they start slowing down?

Answer

1. 23 m/s

e WEB

Can you shoot an object fast enough so that it does not return to Earth? Research escape velocity. Is it the same regardless of the size of an object? How do you calculate it? Write a brief summary of your findings. To learn more about escape velocity, follow the links at www.pearsoned.ca/school/physicssource.

$$\Delta d = \frac{v_f^2 - v_i^2}{2a}$$

$$= \frac{\left(0.00\ \frac{\text{m}}{\text{s}}\right)^2 - \left(10.00\ \frac{\text{m}}{\text{s}}\right)^2}{2\left(-9.81\ \frac{\text{m}}{\text{s}^2}\right)}$$

$$= 5.10\ \text{m}$$

(b) To find the time taken, use the equation $\vec{a} = \frac{\Delta\vec{v}}{\Delta t}$, where $\vec{a}$ is the acceleration due to gravity. Substitute scalar quantities because you are dividing vectors.

$$\Delta t = \frac{v_f - v_i}{a}$$

$$= \frac{0.00\ \frac{\text{m}}{\text{s}} - \left(10.00\ \frac{\text{m}}{\text{s}}\right)}{-9.81\ \frac{\text{m}}{\text{s}^2}}$$

$$= 1.02\ \text{s}$$

Paraphrase

(a) The ball's maximum height is 5.10 m above its launch height.

(b) It takes the ball 1.02 s to reach maximum height.

The next example is a continuation of the previous example: It analyzes the same ball's motion as it falls back down from its maximum height.

Example 1.19

A clown throws a ball upward at 10.00 m/s. Find

(a) the time it takes the ball to return to the clown's hand from maximum height

(b) the ball's final velocity

Given

Consider up to be positive.

$\vec{v}_i = 10.00$ m/s [up] $= +10.00$ m/s

$\vec{a} = 9.81$ m/s² [down] $= -9.81$ m/s²

Required

(a) time taken to land (Δt)

(b) final velocity ($\vec{v}_f$)

Analysis and Solution

(a) For an object starting from rest at maximum height and accelerating downward due to gravity, its motion is described by the equation $\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$, where $\vec{v}_i = 0$ (at maximum height). For downward motion, the ball's displacement and acceleration are in the same direction, so use the scalar form of the equation. For Δd, substitute 5.10 m (from Example 1.18(a)). Rearrange this equation and substitute the values.

Practice Problems

1. A pebble falls from a ledge 20.0 m high.
   (a) Find the velocity with which it hits the ground.
   (b) Find the time it takes to hit the ground.

Answers

1. (a) 19.8 m/s [down]
   (b) 2.02 s

$$\Delta t = \sqrt{\frac{2\Delta d}{a}}$$

$$= \sqrt{\frac{2(5.10 \cancel{m})}{9.81 \frac{\cancel{m}}{s^2}}}$$

$$= 1.02 \text{ s}$$

Compare this time to the time taken to reach maximum height (Example 1.18(b)).

(b) The ball's final velocity (starting from maximum height) when it lands on the ground is

$$\vec{a} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\vec{v}_f = \vec{v}_i + \vec{a}\Delta t$$

$$= 0.00 \text{ m/s} + (-9.81 \text{ m/s}^2)(1.02 \text{ s})$$

$$= -10.0 \text{ m/s}$$

The negative sign means that the direction is downward.

Paraphrase

(a) It takes the ball 1.02 s to return to the clown's hand.

(b) The final velocity at the height of landing is 10.0 m/s [down].

Concept Check

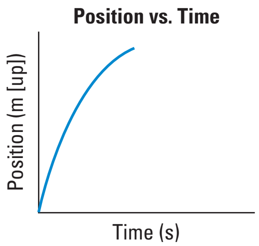
(a) Why does it make sense that the time taken to travel up to the maximum height is equal to the time to fall back down to the starting height?

(b) What variables determine how long a projectile is in the air? Does the answer surprise you? Why or why not?

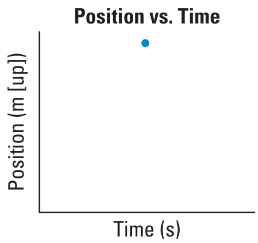
You can use the data calculated in Examples 1.18 and 1.19 to plot a position-time graph of the ball's motion. Because the ball experiences uniformly accelerated motion, the graph is a *parabola* (Figure 1.64).

A Graphical Representation of a Vertical Projectile

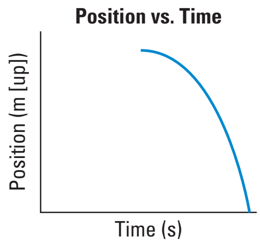
You can now represent the motion of the juggler's ball on a position-time graph. Remember that the ball's motion can be divided into three different stages: Its speed decreases, becomes zero, and then increases. However, the velocity is uniformly decreasing. The graphs that correspond to these three stages of motion are shown in Figure 1.65.



▲ **Figure 1.65(a)** Consider up to be positive. The ball rises until it stops.



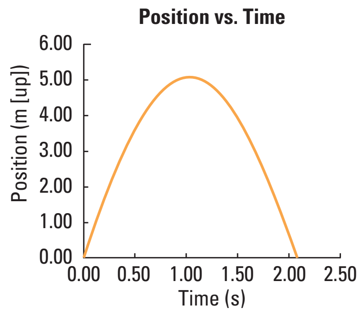
▲ **Figure 1.65(b)** The ball stops momentarily at maximum height.



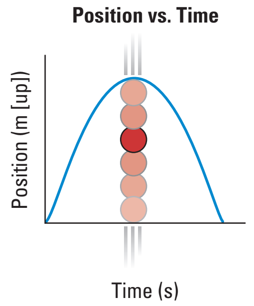
▲ **Figure 1.65(c)** The ball falls back down to its launch height.

info BIT

At terminal velocity, parachuters no longer accelerate but fall at a constant speed. Humans have a terminal velocity of about 321 km/h [down] when curled up and about 201 km/h [down] with arms and legs fully extended to catch the wind.



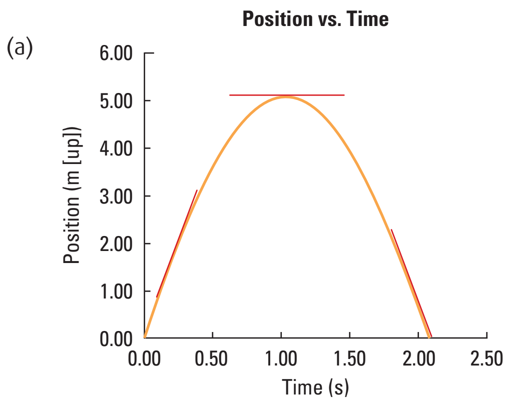
▲ **Figure 1.64** The position-time graph of a ball thrown vertically upward is a parabola.

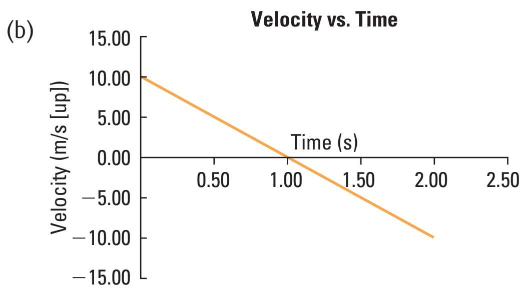


▲ **Figure 1.66** A ball thrown straight up in the air illustrates uniformly accelerated motion.

Now put these three graphs together to generate the complete position-time graph of the ball's motion. Remember that the ball is actually moving straight up and down, and not in a parabolic path (Figure 1.66). Why is the graph of its motion a parabola rather than a straight vertical line?

To generate a corresponding velocity-time graph from the position-time graph in Figure 1.66, draw a series of tangents at specific time instances. Choosing strategic points will make your task easier. The best points to choose are those that begin and end a stage of motion because they define that stage (Figure 1.67(a)).





▲ **Figure 1.67** To generate the velocity-time graph in (b) corresponding to the position-time graph in (a), draw tangents at strategic points.

info **BIT**

In 1883, the Krakatoa volcano in Indonesia hurled rocks 55 km into the air. This volcanic eruption was 10 000 times more powerful than the Hiroshima bomb! Can you find the time it took for the rocks to reach maximum height?

In Figure 1.67(a), notice that the initial slope of the tangent on the position-time graph is positive, corresponding to an initial positive (upward) velocity on the velocity-time graph below (Figure 1.67(b)). The last tangent has a negative slope, corresponding to a final negative velocity on the velocity-time graph. The middle tangent is a horizontal line (slope equals zero), which means that the ball stopped momentarily. Remember that the slope of a velocity-time graph represents acceleration.

Concept Check

What should be the value of the slope of the velocity-time graph for vertical projectile motion?

1.6 Check and Reflect

Knowledge

1. Define a projectile.
2. What determines how long it will take an object to reach the ground when released with an initial velocity of zero?

Applications

3. A student drops a bran muffin from the roof of the school. From what height is the muffin dropped if it hits the ground 3.838 s later?
4. During a babysitting assignment, a babysitter is constantly picking up toys dropped from the infant's highchair. If the toys drop from rest and hit the floor 0.56 s later, from what height are they being dropped?
5. A rock takes 1.575 s to drop 2.00 m down toward the surface of the Moon. Determine the acceleration due to gravity on the Moon.
6. At the beginning of a game, a referee throws a basketball vertically upward with an initial speed of 5.0 m/s. Determine the maximum height above the floor reached by the basketball if it starts from a height of 1.50 m.
7. A student rides West Edmonton Mall's Drop of Doom. If the student starts from rest and falls due to gravity for 2.6 s, what will be his final velocity and how far will he have fallen?
8. If the acceleration due to gravity on Jupiter is 24.8 m/s^2 [down], determine the time it takes for a tennis ball to fall 1.75 m from rest.
9. If a baseball popped straight up into the air has a hang time (length of time in the air) of 6.25 s, determine the distance from the point of contact to the baseball's maximum height.
10. Jumping straight up, how long will a red kangaroo remain in the air if it jumps through a height of 3.0 m?
11. A penny is dropped from a cliff of height 190 m. Determine the time it takes for the penny to hit the bottom of the cliff.
12. A coin tossed straight up into the air takes 2.75 s to go up and down from its initial release point 1.30 m above the ground. What is its maximum height?
13. If a diver starts from rest, determine the amount of time he takes to reach the water's surface from the 10-m platform.
14. A person in an apartment building is 5.0 m above a person walking below. She plans to drop some keys to him. He is currently walking directly toward a point below her at 2.75 m/s. How far away is he if he catches the keys 1.25 m above the ground?

Extensions

15. A rocket launched vertically upward accelerates uniformly for 50 s until it reaches a velocity of 200 m/s [up]. At that instant, its fuel runs out.
    (a) Calculate the rocket's acceleration.
    (b) Calculate the height of the rocket when its fuel runs out.
    (c) Explain why the rocket continues to gain height for 20 s after its fuel runs out.
    (d) Calculate the maximum height of the rocket.
16. A ball is dropped from a height of 60.0 m. A second ball is thrown down 0.850 s later. If both balls reach the ground at the same time, what was the initial velocity of the second ball?

e TEST

To check your understanding of projectiles and acceleration due to gravity, follow the eTest links at www.pearsoned.ca/school/physicssource.

Key Terms and Concepts

kinematics
origin
position
scalar quantity
vector quantity
distance
displacement
velocity
uniform motion
at rest
acceleration
non-uniform motion
instantaneous velocity
tangent
uniformly accelerated motion
projectile motion
projectile
acceleration due to gravity

Key Equations

$$\vec{v} = \frac{\Delta\vec{d}}{\Delta t}$$

$$\vec{a} = \frac{\Delta\vec{v}}{\Delta t} = \frac{\vec{v}_f - \vec{v}_i}{\Delta t}$$

$$\Delta\vec{d} = \frac{1}{2}(\vec{v}_f + \vec{v}_i)\Delta t$$

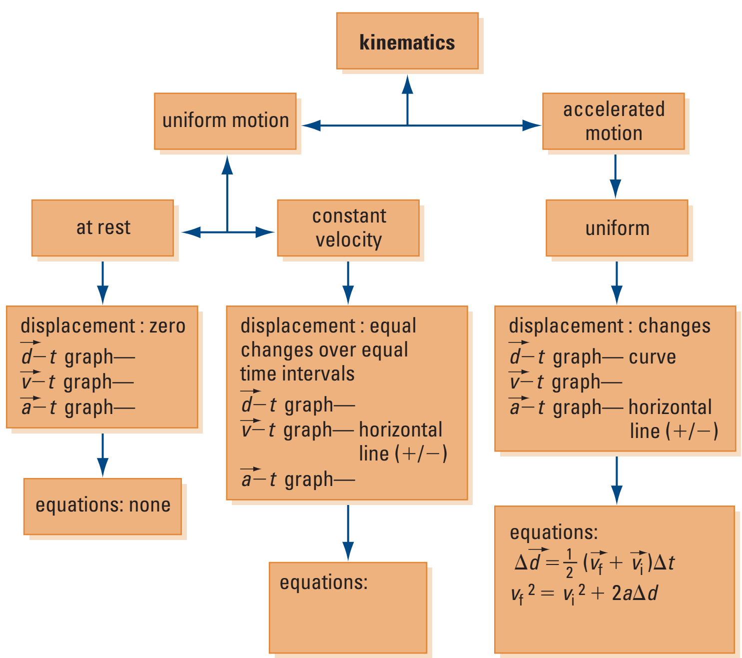
$$\Delta\vec{d} = \vec{v}_i\Delta t + \frac{1}{2}\vec{a}(\Delta t)^2$$

$$\Delta\vec{d} = \vec{v}_f\Delta t - \frac{1}{2}\vec{a}(\Delta t)^2$$

$$v_f^2 = v_i^2 + 2a\Delta d$$

Conceptual Overview

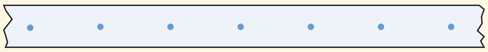
The concept map below summarizes many of the concepts and equations in this chapter. Copy and complete the map to have a full summary of the chapter.



Knowledge

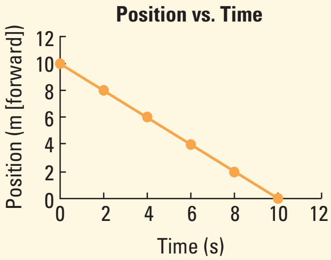
1. (1.1) State two ways in which a vector quantity differs from a scalar quantity. Give an example of each.

2. (1.2) Complete a position-time data table for the motion described by the ticker tape given below.

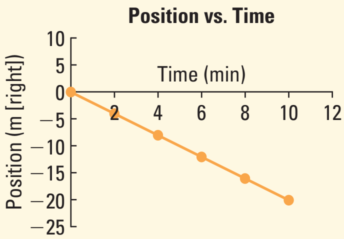


3. (1.3) Determine the velocity of each object whose motion is represented by the graphs below.

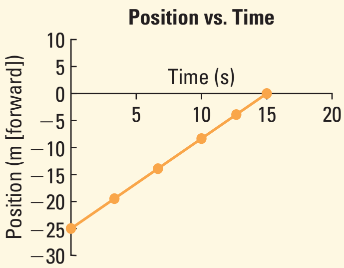
(a)



(b)



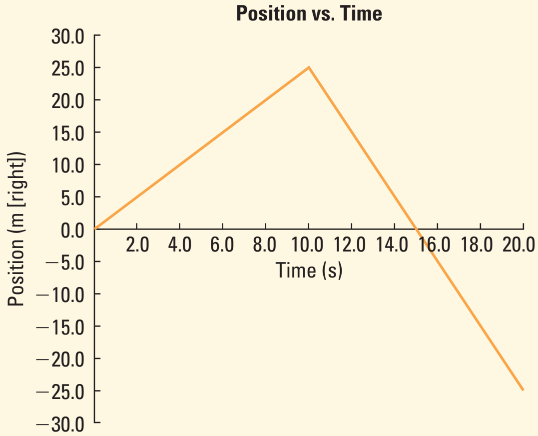
(c)



4. (1.5) What is a vehicle's displacement if it travels at a velocity of 30.0 m/s [W] for 15.0 min?

5. (1.5) How long will it take a cross-country skier, travelling 5.0 km/h, to cover a distance of 3.50 km?

6. (1.2) Determine the average speed, average velocity, and net displacement from the position-time graph below.



7. (1.2) Explain how a person standing still could have the same average velocity but different average speed than a person running around a circular track.

8. (1.6) If an object thrown directly upwards remains in the air for 5.6 s before it returns to its original position, how long did it take to reach its maximum height?

9. (1.6) If an object thrown directly upwards reaches its maximum height in 3.5 s, how long will the object be in the air before it returns to its original position? Assume there is no air resistance.

10. (1.6) What is the initial vertical velocity for an object that is dropped from a height, *h*?

Applications

11. A scuba diver swims at a constant speed of 0.77 m/s. How long will it take the diver to travel 150 m at this speed?

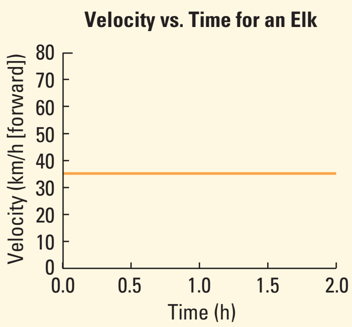
12. In 1980, during the Marathon of Hope, Terry Fox ran 42 km [W] a day. Assuming he ran for 8.0 h a day, what was his average velocity in m/s?

13. Explain how the point of intersection of two functions on a position-time graph differs from the point of intersection of two functions on a velocity-time graph.

14. A thief snatches a handbag and runs north at 5.0 m/s. A police officer, 20 m to the south, sees the event and gives chase. If the officer is a good sprinter, going 7.5 m/s, how far will she have to run to catch the thief?

15. Calculate the magnitude of a bullet's acceleration if it travels at a speed of 1200 m/s and stops within a bulletproof vest that is 1.0 cm thick.

16. From the velocity-time graph below, determine how far an elk will travel in 30 min.



17. The world record for a speedboat is 829 km/h. Heading south, how far will the boat travel in 2.50 min?

18. How much faster is an airliner than a stagecoach if the stagecoach takes 24 h to travel 300 km and the airliner takes 20 min?

19. A car's odometer reads 22 647 km at the start of a trip and 23 209 km at the end. If the trip took 5.0 h, what was the car's average speed in km/h and m/s?

20. A motorcycle coasts downhill from rest with a constant acceleration. If the motorcycle moves 90.0 m in 8.00 s, find its acceleration and velocity after 8.00 s.

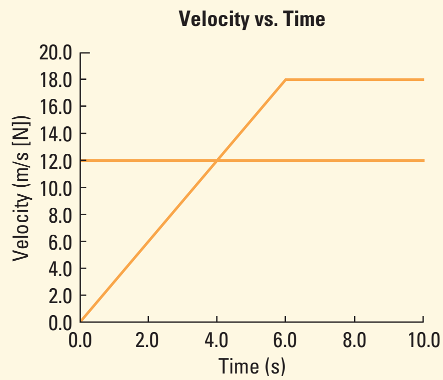
21. A cyclist crosses a 30.0-m bridge in 4.0 s. If her initial velocity was 5.0 m/s [N], find her acceleration and velocity at the other end of the bridge.

22. An object with an initial velocity of 10.0 m/s [S] moves 720 m in 45.0 s along a straight line with constant acceleration. For the 45.0-s interval, find its average velocity, final velocity, and acceleration.

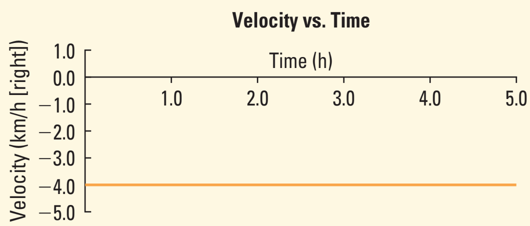
23. During qualifying heats for the Molson Indy, a car must complete a 2.88-km lap in 65 s. If the car goes 60 m/s for the first half of the lap, what must be its minimum speed for the second half to still qualify?

24. A car travelling 19.4 m/s passes a police car at rest. As it passes, the police car starts up, accelerating with a magnitude of 3.2 m/s^2. Maintaining that acceleration, how long will it take the police car to catch up with the speeding motorist? At what speed would the police car be moving? Explain whether or not this scenario is likely to happen.

25. Two cars pass one another while travelling on a city street. Using the velocity-time graph below, draw the corresponding position-time graph and determine when and where the two cars pass one another.



26. Calculate displacement and acceleration from the graph below.

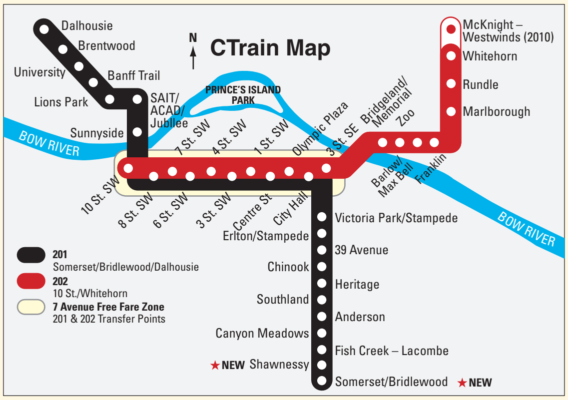


27. Built in Ontario, the world's fastest fire truck, the Hawaiian Eagle, can accelerate at 9.85 m/s^2 [forward]. Starting from rest, how long will it take the Hawaiian Eagle to travel a displacement of 402 m [forward]?

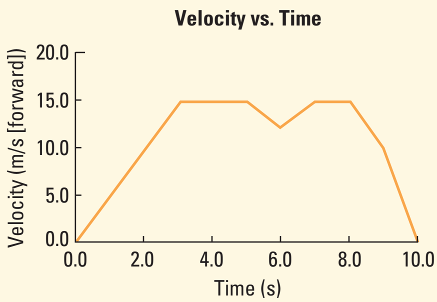
28. A vehicle is travelling at 25.0 m/s. Its brakes provide an acceleration of −3.75 m/s^2 [forward]. What is the driver's maximum reaction time if she is to avoid hitting an obstacle 95.0 m away?

29. Off-ramps are designed for motorists to decrease their vehicles' velocities to move seamlessly into city traffic. If the off-ramp is 1.10 km long, calculate the magnitude of a vehicle's acceleration if it reduces its speed from 110.0 km/h to 60.0 km/h.

30. Calgary's CTrain leaves the 10 St. S.W. station at 4:45 p.m. and arrives at 3 St. S.E. at 4:53 p.m. If the distance between the two stops is 3.2 km, determine the CTrain's average velocity for the trip.



31. Describe the motion of the truck from the velocity-time graph below. When is the truck at rest? travelling with a uniform velocity? When is its acceleration the greatest?



32. A racecar accelerates uniformly from 17.5 m/s [W] to 45.2 m/s [W] in 2.47 s. Determine the acceleration of the racecar.

33. How long will it take a vehicle travelling 80 km/h [W] to stop if the average stopping distance for that velocity is 76.0 m?

34. The Slingshot, an amusement park ride, propels its riders upward from rest with an acceleration of 39.24 m/s^2. How long does it take to reach a height of 27.0 m? Assume uniform acceleration.

35. Starting from rest, a platform diver hits the water with a speed of 55 km/h. From what height did she start her descent into the pool?

36. A circus performer can land safely on the ground at speeds up to 13.5 m/s. What is the greatest height from which the performer can fall?

37. A contractor drops a bolt from the top of a roof located 8.52 m above the ground. How long does it take the bolt to reach the ground, assuming there is no air resistance?

38. An improperly installed weathervane falls from the roof of a barn and lands on the ground 1.76 s later. From what height did the weathervane fall and how fast was it travelling just before impact?

39. Attempting to beat the record for tallest Lego structure, a student drops a piece from a height of 24.91 m. How fast will the piece be travelling when it is 5.0 m above the ground and how long will it take to get there?

Extension

40. Weave zones are areas on roads where vehicles are changing their velocities to merge onto and off of busy expressways. Suggest criteria a design engineer must consider in developing a weave zone.

Consolidate Your Understanding

Create your own summary of kinematics by answering the questions below. If you want to use a graphic organizer, refer to Student References 4: Using Graphic Organizers on pp. 869–871. Use the Key Terms and Concepts listed on page 64 and the Learning Outcomes on page 4.

1. Create a flowchart to describe the changes in position, velocity, and acceleration for both uniform and accelerated motion.
2. Write a paragraph explaining the two main functions of graphing in kinematics. Share your report with another classmate.

Think About It

Review your answers to the Think About It questions on page 5. How would you answer each question now?

eTEST

To check your understanding of motion in one dimension, follow the eTest links at www.pearsoned.ca/school/physicssource.